DOCUMENT RESUME

ED 094 990 SE 017 530

AUTHOR	Pedoe, Daniel; Pedoe, Mary
TITLE	Mathematics for the Elementary School, Unit 11, Numeration.
INSTITUTION	Minnesota Univ., Minneapolis. Minnesota School Mathematics and Science Center.
SPONS AGENCY	National Science Foundation, Washington, D.C.
PUB DATE	65
NOTE	136p.
EDRS PRICE	MF-$0.75 HC-$6.60 PLUS POSTAGE
DESCRIPTORS	Activity Learning; Addition; Curriculum; *Elementary School Mathematics; *Inequalities; Instructional Materials; Number Concepts; *Number Systems; Subtraction; *Teaching Guides; Units of Study (Subject Fields); Whole Numbers; Worksheets
IDENTIFIERS	MINNEMAST; *Minnesota Mathematics and Science Teaching Project; Number Line; Properties (Mathematics)

ABSTRACT

The Minnesota School Mathematics and Science Teaching (MINNEMAST) Project is characterized by its emphasis on the coordination of mathematics and science in the elementary school curriculum. Units are planned to provide children with activities in which they learn various concepts from both subject areas. Each subject is used to support and reinforce the other where appropriate, with common techniques and concepts being sought and exploited. Content is presented in story fashion. The stories serve to introduce concepts and lead to activities. Imbedded in the pictures that accompany the stories are examples of the concepts presented. This unit stresses the geometric interpretation of number on the number line. The notions of "greater than," "less than" and "between" are presented (or reviewed) by reference to the number line. Egyptian and Roman numeration systems are explored and an intuitive presentation of non-decimal systems is contained in some of the activities. Worksheets and commentaries to the teacher are provided and additional activities are suggested. (JP)

MINNE

Math And Science Teaching project

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UNIT XI

NUMERATION

MINNESOTA SCHOOL MATHEMATICS AND SCIENCE CENTER 1968

MATHEMATICS
FOR THE
ELEMENTARY SCHOOL

UNIT XI
Numeration

The Minnesota School Mathematics and Science Teaching Project
produced these materials under a grant from the
National Science Foundation

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We are deeply indebted to the many teachers who

used earlier versions of this material

and provided suggestions for

this revision

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NUMERATION

The purpose of this unit is to extend the concept of number by utilizing the number line.

The specific purposes are:

1. To show that the number line is a geometric association of a point on the line and a number.
2. To introduce the meaning of "positive integer".
3. To review the number scale.
4. To develop an understanding of the relationship of positive integers or combination of integers as:
   a. shown by comparison in determining which is "greater than".
   b. an aid to estimating possible sums as illustrated with the work in the part called "between".
5. To teach that the concept of number may be and has been expressed in written form in many ways.

This unit of work is intended to create an "awareness" of these objectives. It is not expected that the teacher will need to develop many skills.

Materials:

Individual story booklets -- "A Story About Numerals" for each child

Drawing paper 8 1/2" x 11"

Pencils or crayons for pupils

Purpose:

To create an awareness of the many ways numerals help if one knows how to read and interpret them.

Procedure:

Many things help to make our lives more pleasant each day. Numerals have a way of helping us. Some numerals helped me today. Did any help you? Tell us about it.

We find numerals at home, at school, and out in other places. They can serve us well if we know how to read them and to understand what they mean.

I have a delightful picture booklet for you, today. It is called "A Story About Numerals." It reminds us of some of the ways numerals serve us. As we read it think of any other way numerals help.

Read the story.

Possible follow-up:

1. Discuss other ways numerals help. Let children illustrate their ideas and write the legend.

THE NUMBER SCALE

The purpose of this chapter is to provide a review of the continuous number scale from 0-100.

The Minnemast number line should be posted for the children's use.

An approach to ordered numbers can be made by teaching a recognition of patterns and relationships that exist between numbers.

The pattern in this set is readily recognized by children [8, 9, 10, 11] while this pattern [12, 15, 18] may hold more of a challenge.

A "2-Chart", "3-Chart", and "5-Chart" are developed by the class to show relationships between numbers and to discover patterns.

The class should be encouraged to write and recognize other patterns.

Some discoveries to make on the number charts are:

1	2
3	4
5	6
7	8
9	10

1. How much greater is any integer than one that goes just before it on the number line? (one)
2. What kind of integers are in column one? (odd)
   Column two? (even)
3. If you add any two numbers in column 1, in what column will you find the sum?
   Did you add odd or even numbers? Was the sum odd or even?
4. Add a number from column 1 and a number from column 2, and tell what you discover.
5. Add two numbers from column 2 and find what happens.

What interesting discoveries can you make on the 3-Chart and the 5-Chart?

Procedure:

Write a set of numerals on the chalkboard. For example:

20, 21, 22, 23, 24, 25

Then proceed to question the class:

1. Are these integers in counting order?
2. How can we decide?
3. Is 20 $>$ 21?
4. Is 21 $>$ 22?

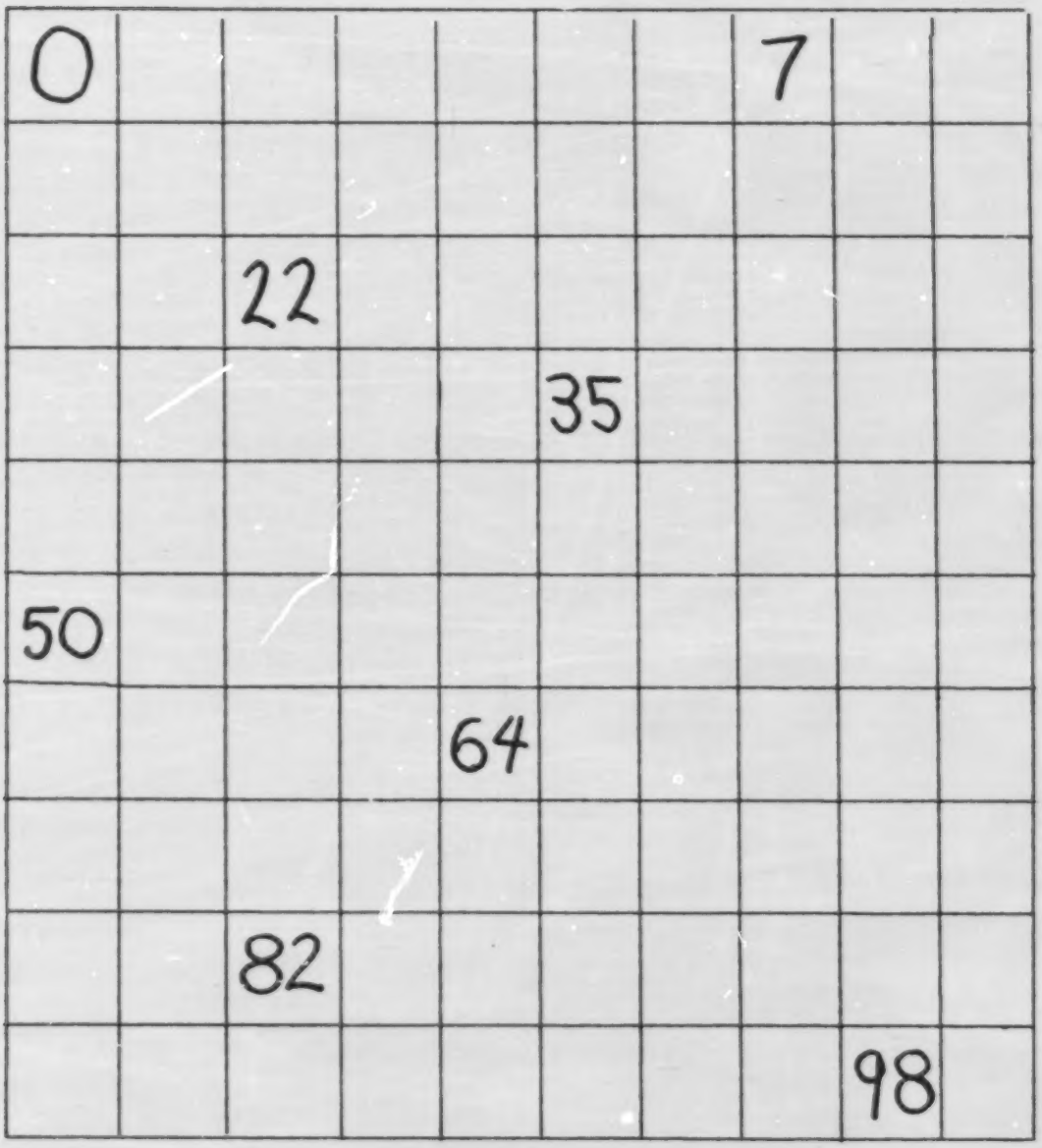
Compare two integers at a given time until they are ordered.

Worksheet 1

THE COUNTING NUMBERS 0-99

Name:_____

Start at zero and work to the right. Fill each blank square with the next counting number.



Worksheet 2

THE COUNTING NUMBERS 1-50

Start at 1.

Name: _____

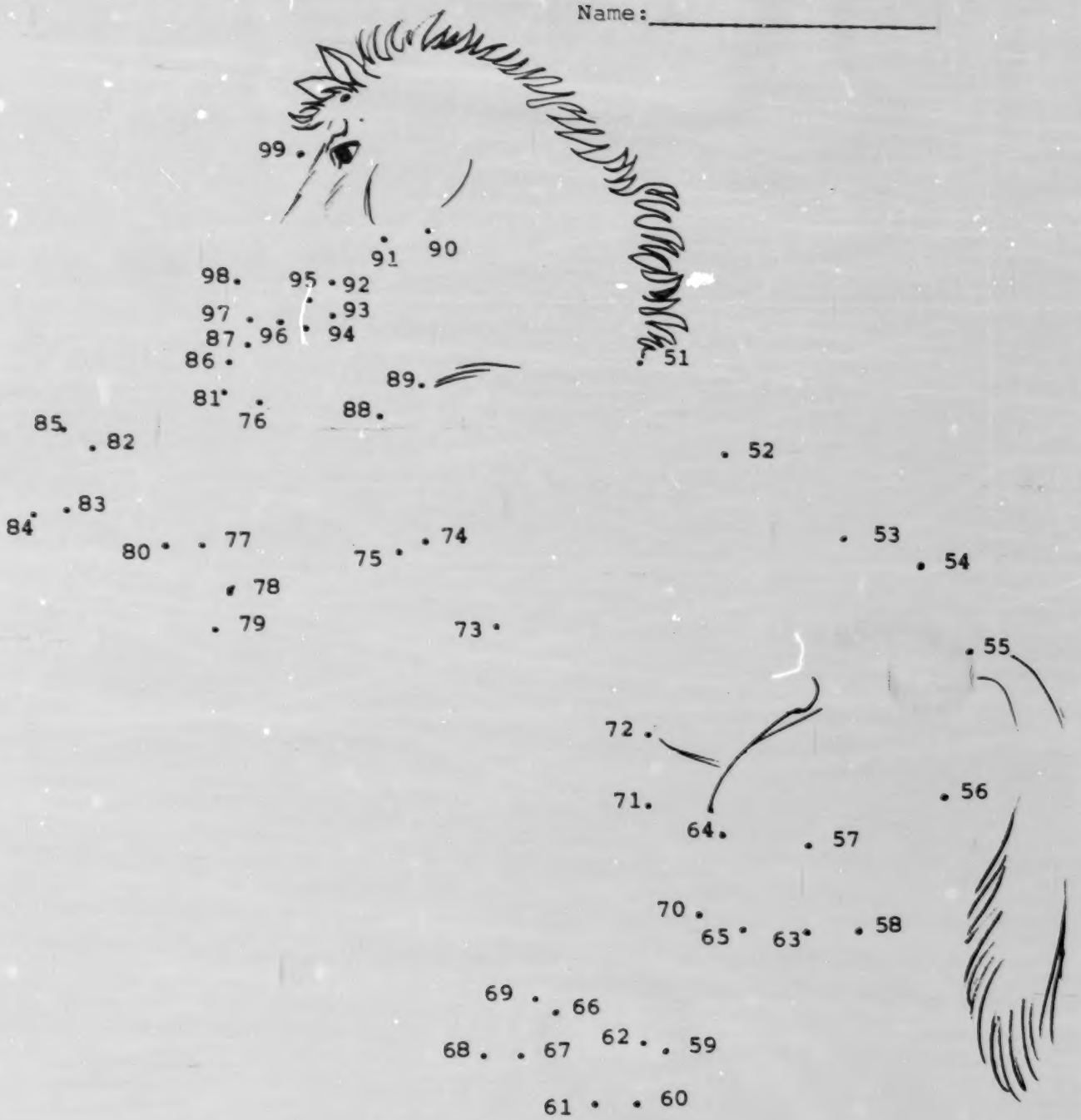
Draw a line from dot to dot, from one numeral to the next in counting order.

See what picture you have made!

Worksheet 3

THE COUNTING NUMBERS 50-99

Name:_____



Draw a line from dot to dot, from one numeral to the next in counting order.

See what picture you have made!

CHARTS HELP TO DISCOVER RELATIONSHIPS

Procedure:

I have started a chart on some oaktag. It is a number chart.

2-Chart

1	2
3	4
5	6
7	8
9	10

1. Tell me what to write in the blank spaces.
2. Now look at our chart:

a. How much greater is the integer in the right column than the one in the left?

b. Compare two integers in left column. How much greater is one than the other? In the right column?

c. What about the numbers in the first column? (odd)

d. The second? (even)

e. What if we add an odd number and another odd number, what kind of a number do we get? I shall write a note for us:

odd number + odd number = even number

f. What if we add an even number and an even number?

even number + even number = even number

g. What other combination can we make? Yes - what happens?

odd number + even number = odd number

When you add numbers, this discovery should help you tell if your sums could be right?

h. Compare the two numbers in the first column. What do you see?

i. Does this relationship continue all the way down the column?

j. Is it true for the second column?

COUNTING BY TWO'S

Sometimes it is very handy to be able to count by two's.

When we empty our piggy banks we often count pennies in a quick way by putting two aside at a time.

We might count how many children we have in the room by two's.

On the sheet of paper I shall give you, count by two's. Fill as many of the blank spaces as you can.

Worksheet 4

USING THE 2-CHART Name:_____

Think of a number 2 greater than the one before.

Fill as many blank ☐ 's as you can.

Use the 2-Chart.

2	4			

Worksheet 5

ADDITION ON THE 2-CHART Name:_____

Use the 2-Chart.

Add these numbers.

Then fill in the rules.

2 + 6 = _____ 4 + 4 = _____ 8 + 2 = _____	3 + 3 = _____ 5 + 5 = _____ 7 + 3 = _____
When we add even numbers and even numbers, we get _____ numbers.	**When we add odd numbers and odd numbers, we get _____ numbers.**

3 + 4 = _____

3 + 6 = _____

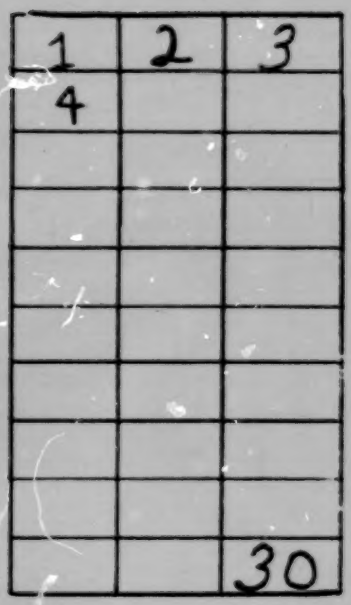
7 + 2 = _____

When we add odd numbers and even numbers, we get _____ numbers.

CHARTS HELP TO DISCOVER RELATIONSHIPS

Procedure:

Let us make a chart with 3 numerals in a row.



1. Tell me what numerals to write.
2. Look at these columns. Do you see anything about the numerals in them?
3. Let us consider column 3.
   Read these numerals to the class....
4. Do you think people might ever have to count by 3's?

Some children have enjoyed playing a game called "Buzz". This is how it is played:

The first person says --- "1"

The second person says --- "2"

But the third person cannot say "3". He must say "Buzz".

Look at our chart. The forbidden numerals are in the third column. For the first 3, we say "Buzz!" --- but for the 6, we say "Buzz! Buzz!" Who knows why?

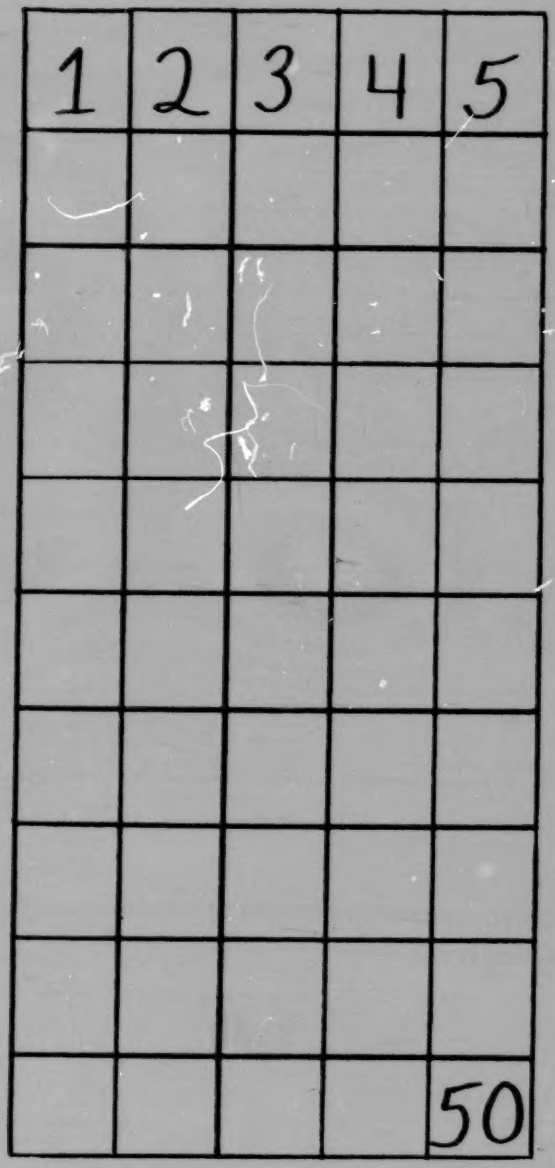
If you make a mistake and call 6 instead of "Buzz! Buzz!" you are out of the game.

Let us practice with the chart once, before we play.

BUZZ GAME

1	2	1) Buzz
4	5	1) Buzz 2) Buzz
7	8	1) Buzz 2) Buzz 3) Buzz
10	11	Buzz 4 Times
13	14	Buzz 5 Times
16	17	Buzz 6 Times
19	20	Buzz 7 Times
22	23	Buzz 8 Times
25	26	Buzz 9 Times
28	29	Buzz 10 Times

CHARTS HELP TO DISCOVER RELATIONSHIPS



1. What kind of a chart have I started on the board today?
2. What can we learn about the columns of numbers?
3. Count by 5's to 50.
4. A game to play -- Have a child hide some object. As the child is hiding it, the class counts by 5's to 50. Choose children to find the object while the class counts by 5's from 0-100.

Worksheet 6

SEEING A PATTERN

Name:_____

Look at the set.

What pattern do you see?

Put a numeral in the ☐ which will carry out the pattern.

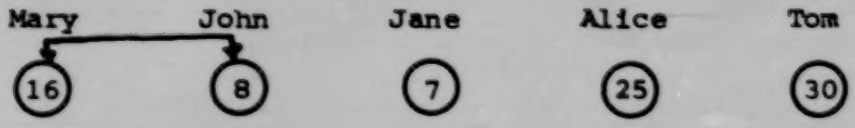
2, 4, 6, ☐	☐, 6, 9, 12
20, ☐, 30, 35	25, 30, ☐, 40
10, ☐, 14, 16	82, 84, 86, ☐
10, 20, 30, ☐	80, 85, ☐, 95
☐, 27, 30, 33	18, ☐, 24, 27

ADDITIONAL ACTIVITIES

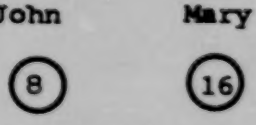
Ordering of Numbers

1. On slips of paper have the children write their name and the date of the month on which they were born.

Collect them and select 5 children whose dates differ to line up before the class. (The children are <u>not</u> given their slips of paper.) They are told that their birthdate falls on a date from 1-31. They are not standing in the proper order of their birthdates. Ask them to quickly rearrange themselves in this order. How can they do it? There is only one possible way of doing this. <u>Two adjacent numbers must be considered at a time</u> like this:



Mary and John decide which date comes first. They rearrange themselves.



Then Mary and Jane consider the problem and rearrange themselves.

This continues until the row is ordered.

2. Do the same as the above, but race one group against another.
3. Use the children's weight. This involves higher numbers.

4. Have the class draw random numbers to reorder.

5. Can you play a game like the above by giving children a large card with a numeral on it? Have a captain rearrange the group with the least possible moves. The chairs could be arranged in the setting for the game. Keep a record of the captain's number of moves. Let different children play this role. Who was the best captain?

WE WORK WITH NUMBERS BUT WRITE NUMERALS

Materials:

flag, apple, book, or other concrete objects

To develop that:

numbers are ideas for which symbols can be written.
these symbols are called numerals.

Background:

Numbers are ideas. They do not exist physically, but are the abstractions in our mind.

Number does not imply consideration for a property of an object. It does not consider size, shape, or texture.

The idea of number is expressed by a symbol called a numeral. We write numerals, but we think and manipulate mentally with numbers. For example, the numeral 4 characterizes a whole class of sets, each of which has 4 members.

Procedure:

Have flag, apple, and other concrete objects on display.

Can you find something in the room that is a symbol for our country?

Is our flag our country?

But when we see it, we are reminded of our country. The flag is a symbol. We think of what it means when we give the pledge of allegiance, don't we?

Teacher prints "apple" on the board.

Find an apple for me. Is there any other apple in the room?
Is this an apple on the board? What is it?
It wouldn't be very tasty to eat the apple which appears on the board, would it?

Do you know any other symbols?
If I put this symbol [BRAVE] on the board, what idea do you get? We have the idea of number in mathematics for which we want to write a symbol.
What idea do you get if I say "eight", "five", "three"?
Come to the board and write a symbol for eight, five, three.
These symbols which you have written have a name. They are called numerals.
Let us keep in mind that marks we make to express a number idea are really just numerals and not numbers, no more than the word "apple" on the board is a real apple to eat.

Today I would like to have you show me how well you can write the numerals from 0 to 50.
Those that have time and wish to do so may write the numerals from 50 to 100.

A

STORY ABOUT

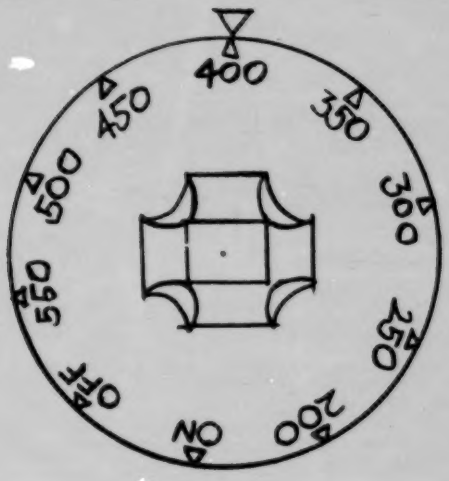
NUMERALS

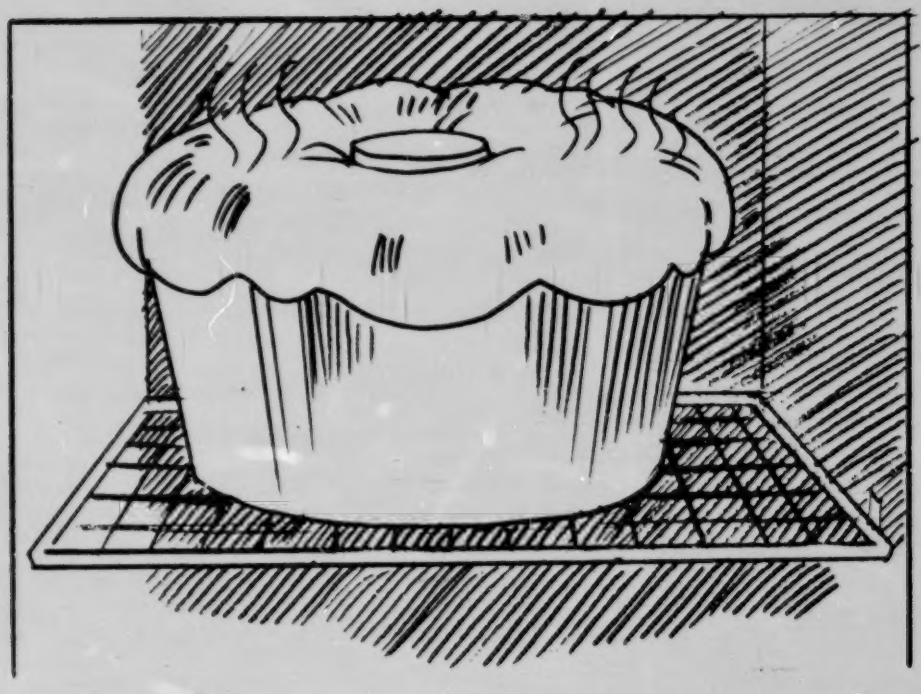
A clock has numerals so you can tell time.

People in the pet store put numerals on animal cages.

Jackets have numerals in them to show the size.

Dials on telephones have numerals on them so you can dial 373-3522.





Oven controls have numerals.

City houses have numerals on them to help the mailman.

MANY WAYS TO WRITE A NUMBER

Materials:

Story Booklets, "Here and There Adventures"

To develop that:

1. the symbol for writing a number was invented by early man to meet his needs.
2. the Egyptians used symbols which differed from ours but seemed to serve their purpose.
3. the Roman symbols are still in limited use today.
4. a number may be written or expressed in many ways.

Procedure:

Man used many kinds of symbols to express number. Here are some booklets telling about some of these. Which of these number symbols were really used by people?

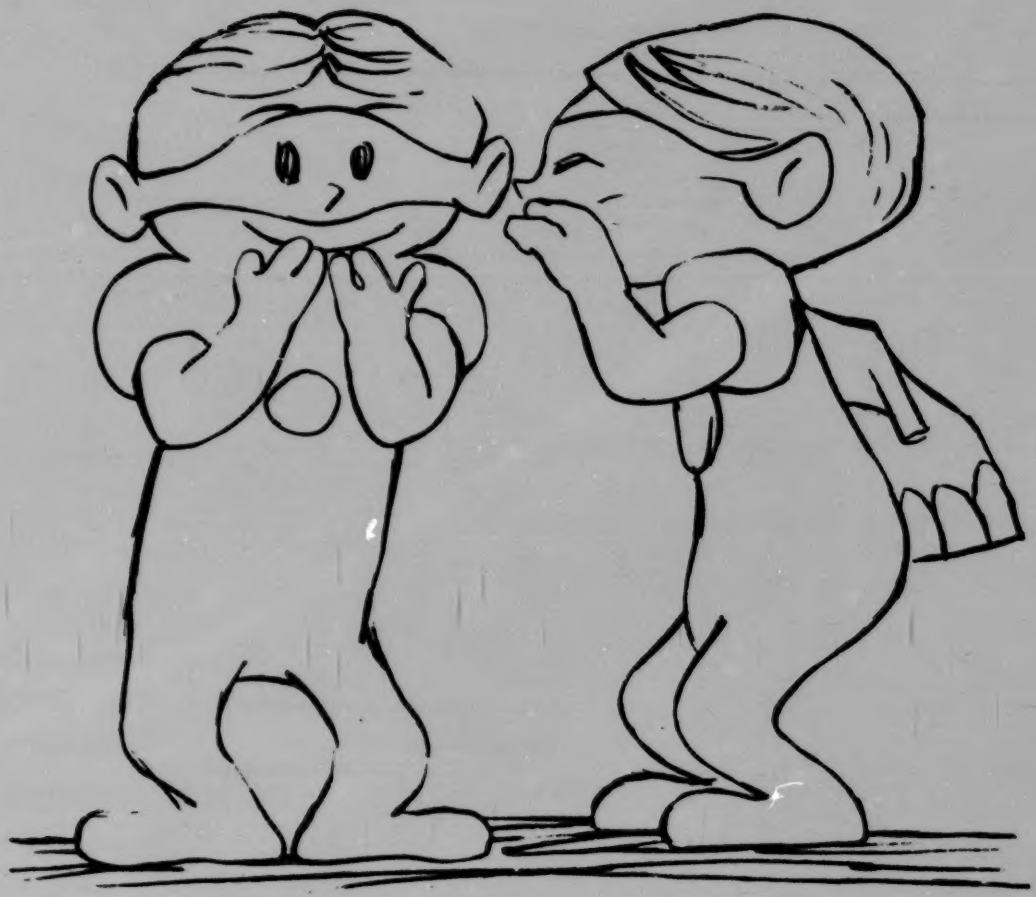
NOTE: Follow each reading with the corresponding exercise.

HERE AND THERE ADVENTURES

PART I

THE TAOTIAN BOY

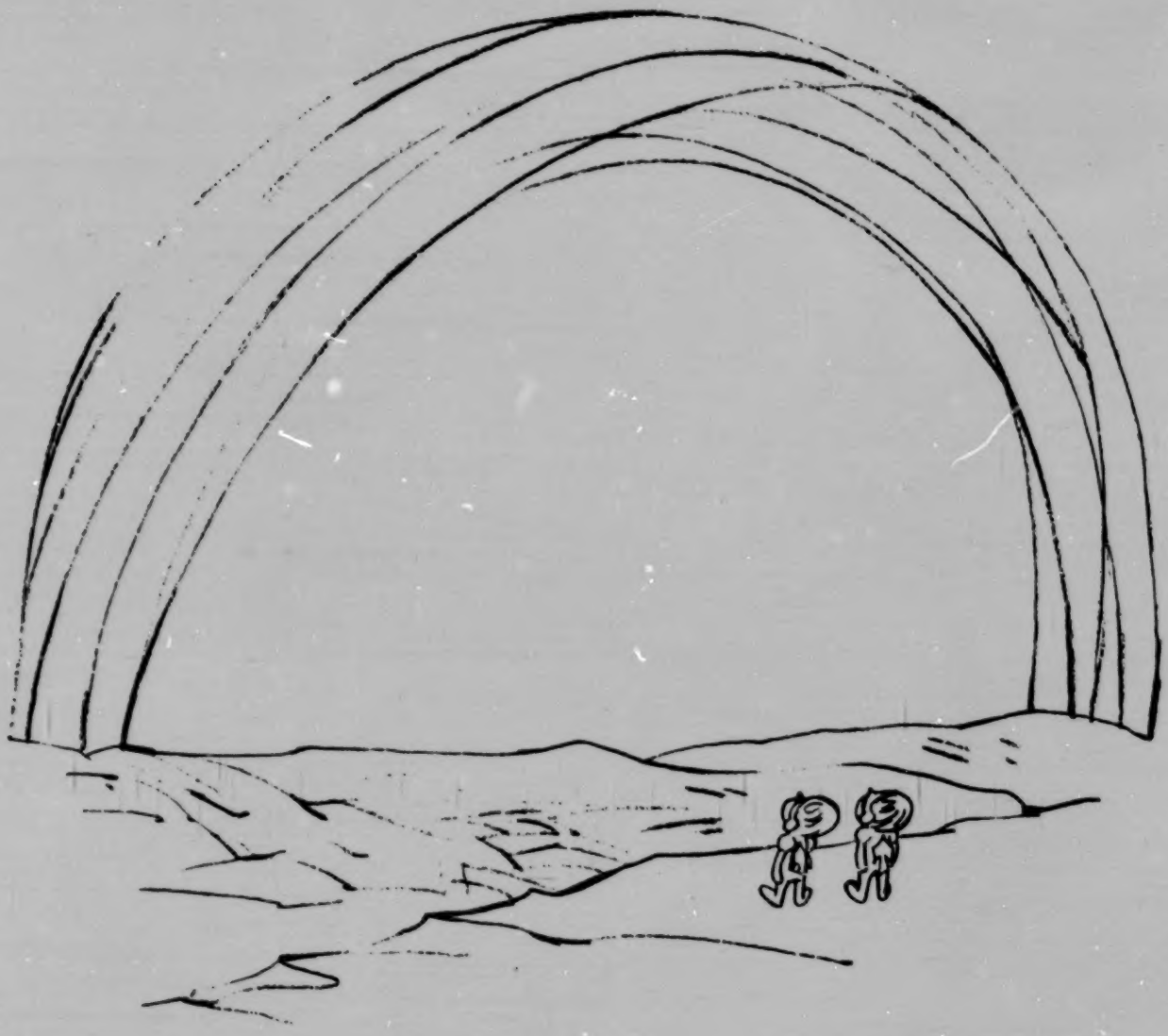
(PRONOUNCE: TAY OH SHUN)



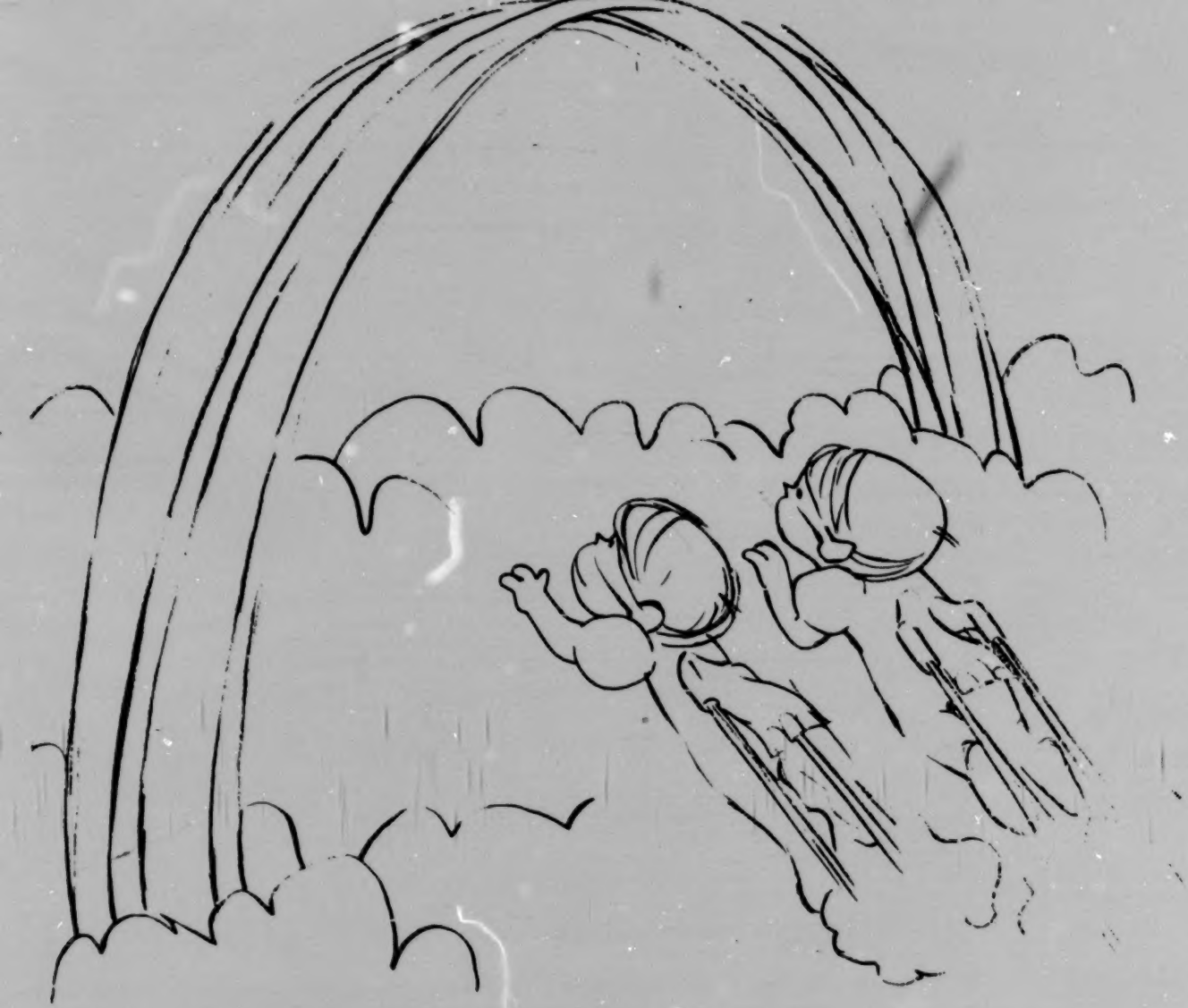
Once there were two Space boys who lived on a planet called Taos. They were called Taotians.

The Space boys learned to walk and talk and fly about like every other Taotian. They learned to take long and short flights together through space. What adventures they had!

They enjoyed hearing the thunder and seeing the streaks of lightning.



One day there was a very heavy shower of rain. There was loud thunder and sharp lightning. There were big crystal looking raindrops. After all this, there was a huge, colorful rainbow.



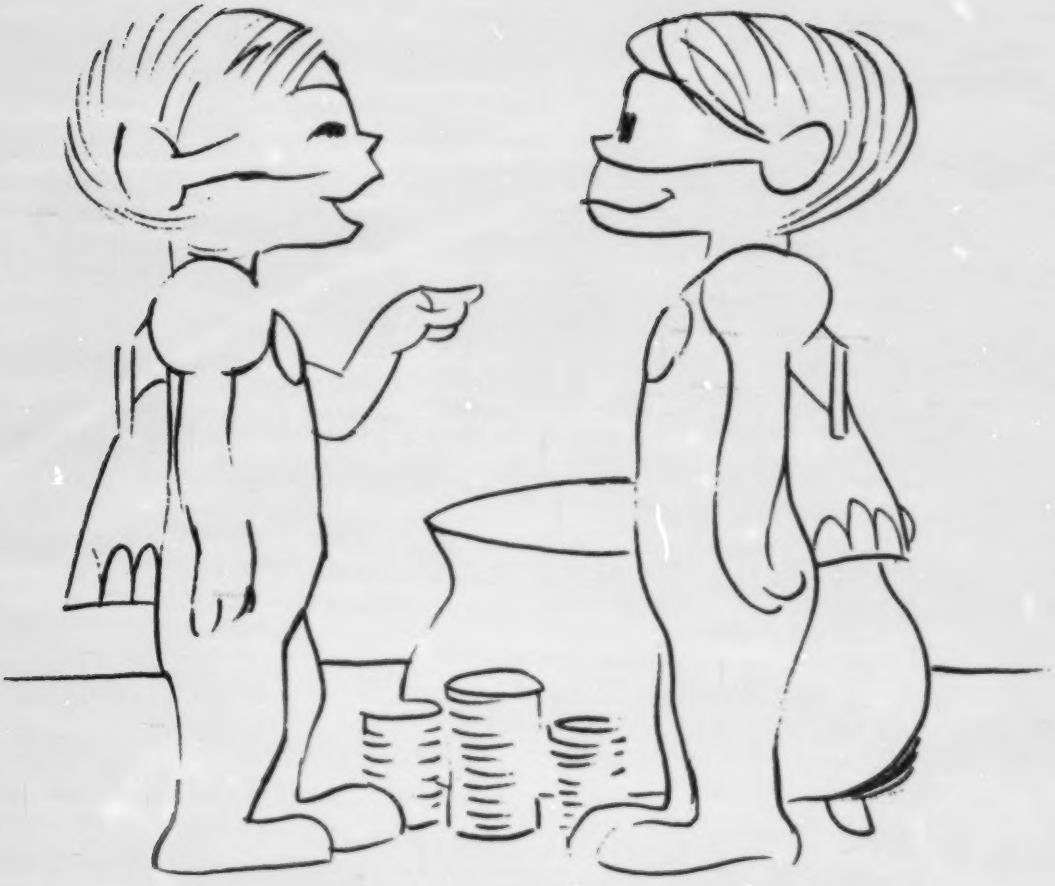
The Taotians made a flight to the top.

The two friends walked around the colorful arch. They were admiring the puddle-made patches which they saw far down below. Just then they stepped on a slippery spot and lost their balance.

Down the opposite side of the curve they slid.

They rode on the curve of colors all the way to the bottom where they slid head first into a pot of golden coins.

The Taotian boys sat at the end of the rainbow counting the pieces of gold in their treasure. There were so many pieces that the boys began to get mixed up in their counting.



The one said', "You keep our treasure safe and I will find something on which to write our numerals. Then we will know exactly how many pieces of gold we have."

After a time he flew back with a piece of cloud. He said, "This will be good to write our numerals on because it is so white."

"I like that," said the other, "but what will we write with?"

"Oh, that will be easy," said the one. "We can use the wet colors of the rainbow. We can dip our fingers in them and paint our numerals on the cloud."

They decided to make up their own way to write numerals. They decided to use pictures of things they knew about for their numerals.

The one said, "I will count the coins if you will paint what they mean in numeral pictures."

He began to count. He said, "One," and then he watched for his friend to paint a numeral on the cloud piece.

The other dipped his pointing finger into the colorful paint. He repeated, "One." Then he said, "I make this circle picture. It is the shape of the sun. I use it because there is one sun in the sky just like there is one gold coin in your hand."

The one said, "I understand," and then he counted more coins. He said, "2, 3, 4."

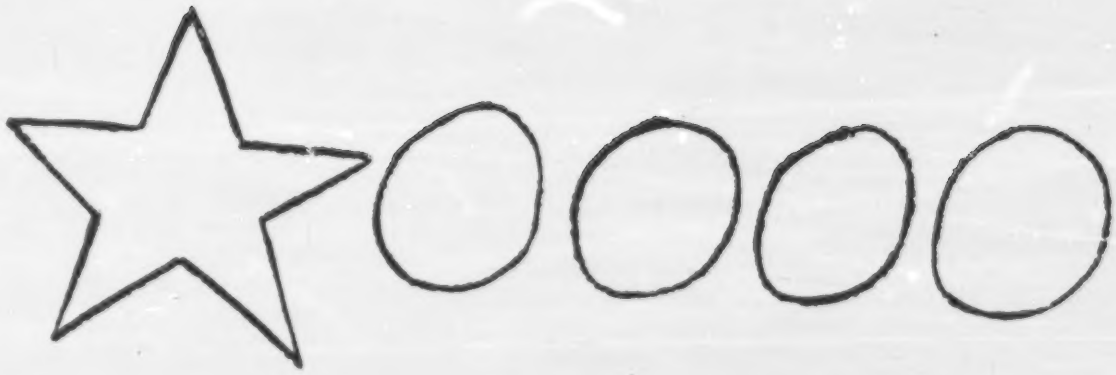
The other made more suns. He said, "Now I have four circles just like you have four coins in your hand."

The other repeated, "Five," and then he said, "I make this star picture for number 5 because a star often has five points just like you have five shiny coins in your hand."

A star and a sun were painted together for the number six because the star meant five and the sun meant one more.

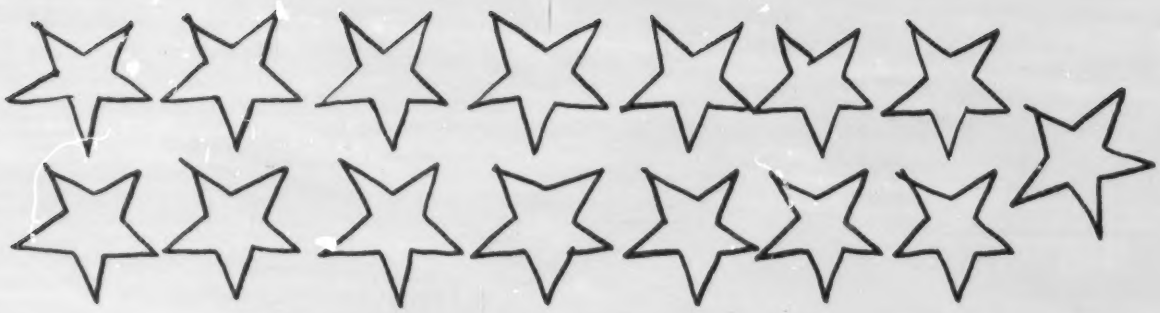
Seven became a star and two suns.

Eight was a star and three suns and . . .



nine became a star and four suns.

When the one counted, "Ten," the other said, "I'll paint two stars for the number ten, one over the other.



After all the coins had been counted and the picture painting had been done, the cloud had seven double stars and one star all in a line.

The Taotians counted the number pictures to see how many coins they owned. This is what they said as they counted the painted cloud pictures: "10, 20, 30, 40, 50, 60, 70 and one star makes five more. We have 75 pieces of gold."

Worksheet 7

MY OWN SYMBOLS FOR NUMBERS Name:_____

Think of some other ways of writing numbers:

What symbol would you use for:

one _____ five _____ ten _____

Now write these numerals:

4 _____

8 _____

12 _____

23 _____

BE READY TO EXPLAIN YOUR WAY TO US.

Worksheet 8

MY OWN CLOCK

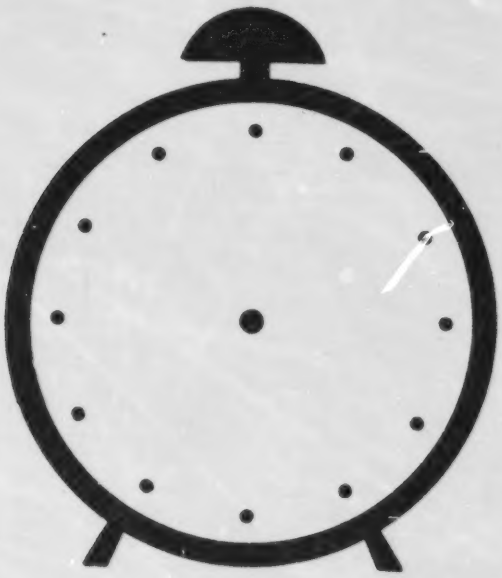
Name:_____

Think of some other ways of writing numbers.

What symbols would you use for:

one_____five_____ten_____

Make a clock using your symbols. We will read the time the clock hands show.



Worksheet 9

SOME WAYS TO USE MY SYMBOLS Name:_____

Think of some other ways of writing numbers.

What symbol would you use for

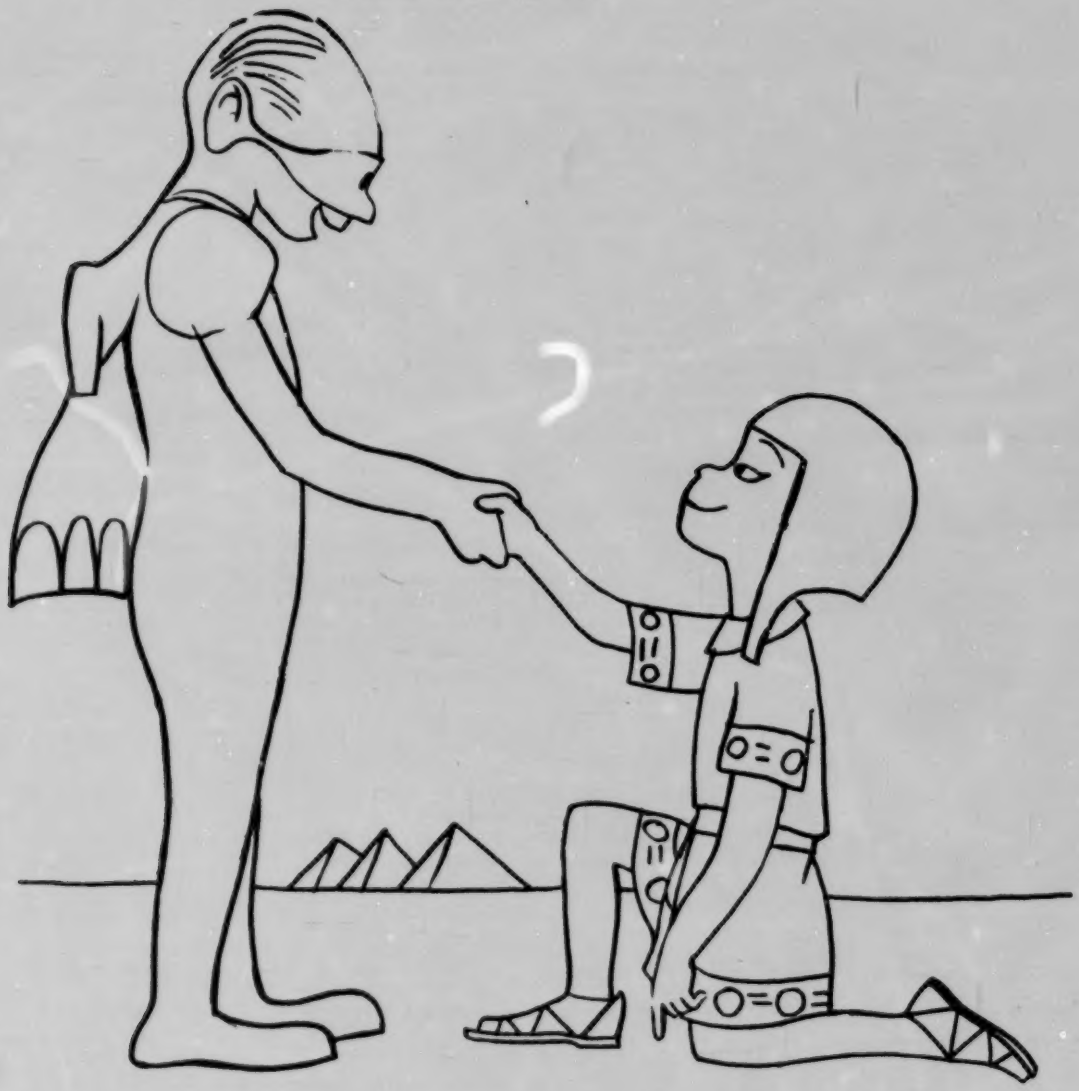
one_____ five_____ten_____

Make a picture for a class book showing how your symbols could be used.

PART II

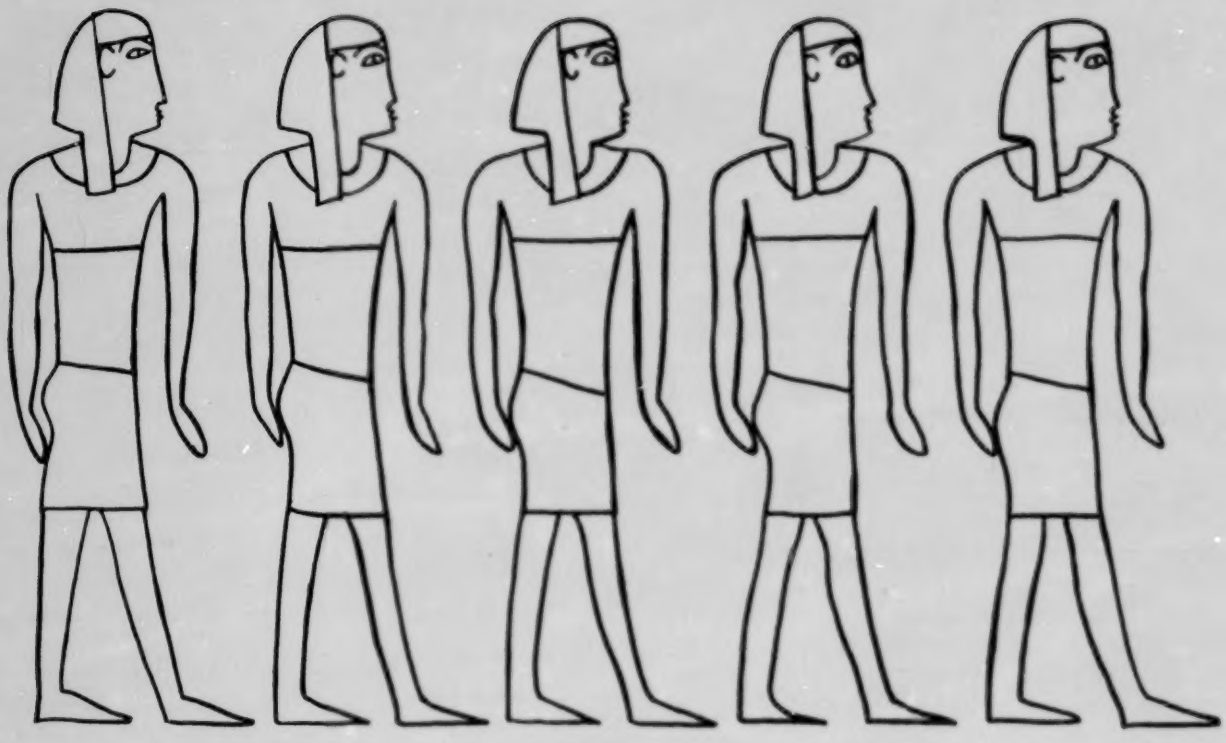
THE EGYPTIAN BOY

An old Space man told the boys that he hadn't been to Earth for years and years. He also told them that when he was there he met children from three different countries. This is the story that he told: One time I met a Babylonian boy.

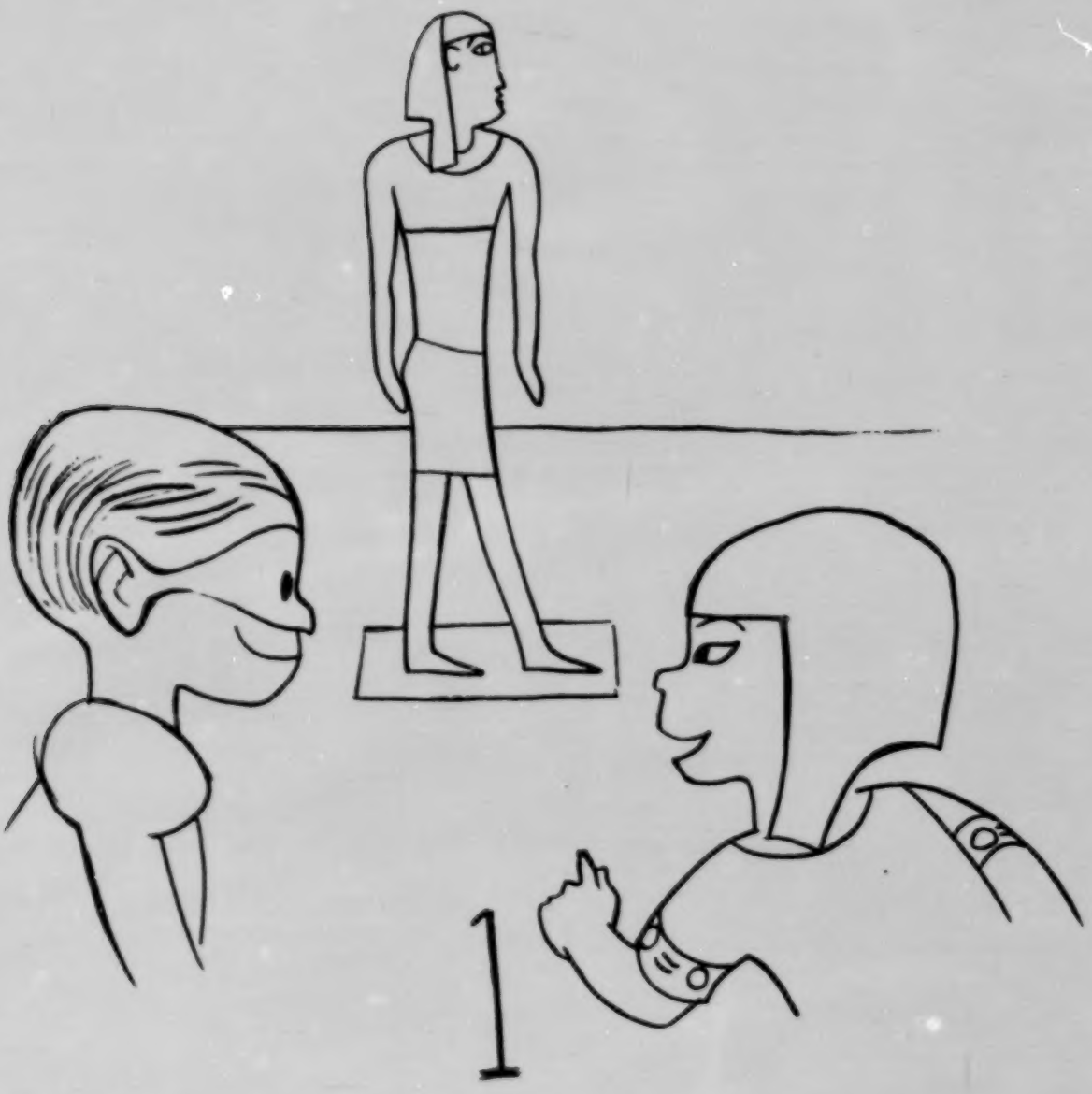


Another time I met an Egyptian boy sitting at a sandy road side. He was scratching his kind of numerals in the sand.

"What are you counting?" I asked.



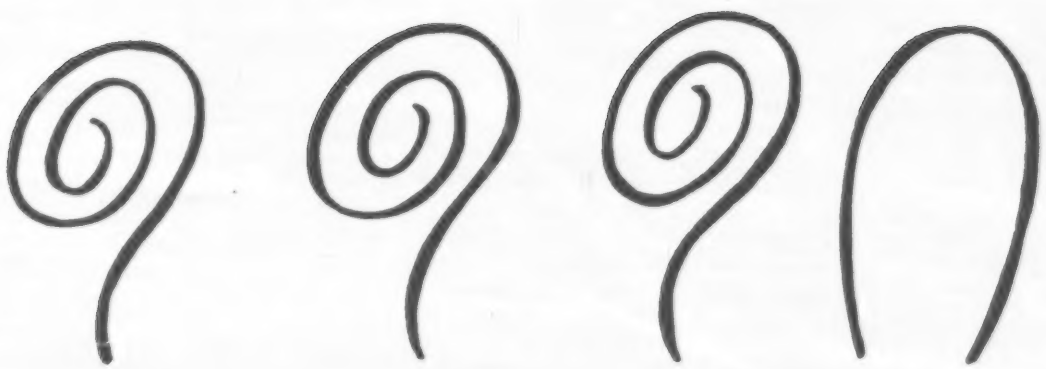
"I am counting all those men who are marching away to work on the pyramid."



For every one man that went by I put "1."

For every ten men that went by I made a 𓎆 .

And for every hundred men that went marching by, I made a 𓍢 .



I have my kind of numerals in this picture:
𓏺𓏺𓏺𓎆. Can you tell how many men went marching by?

"I counted the pictures. This is what I said, '100, 200, 300 and ten more.' That makes 310 in all. The Egyptian boy told me I was right."

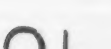
Worksheet 10
EGYPTIAN HIEROGLYPHICS

Name:_____

I. Can you read these?

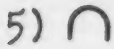
Key: 1 = 1 ∩ = 10 @ = 100

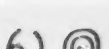
1) = _____

2)  = _____

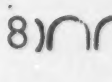
3) = _____

4) = _____

5)  = _____

6)  = _____

7) = _____

8)  = _____

II. Find the answer. Write it in Egyptian hieroglyphic

1) = _____

2) = _____

3) = _____

4) = _____

5) = _____

6) = _____

Additional Activities

1. A child may write a numeral using Egyptian hieroglyphics. He calls on someone to read his numerals.

2. The teacher may write 3 such numerals on the board. Call a number. Have a child circle the proper symbol.

3. Place some blank cards out for the children's use. Have those that wish to do so make finding a sum or missing addend using Egyptian hieroglyphics.

4. Make a class chart.

Numerals		
Arabic	Egyptian	Save for Roman

PART III

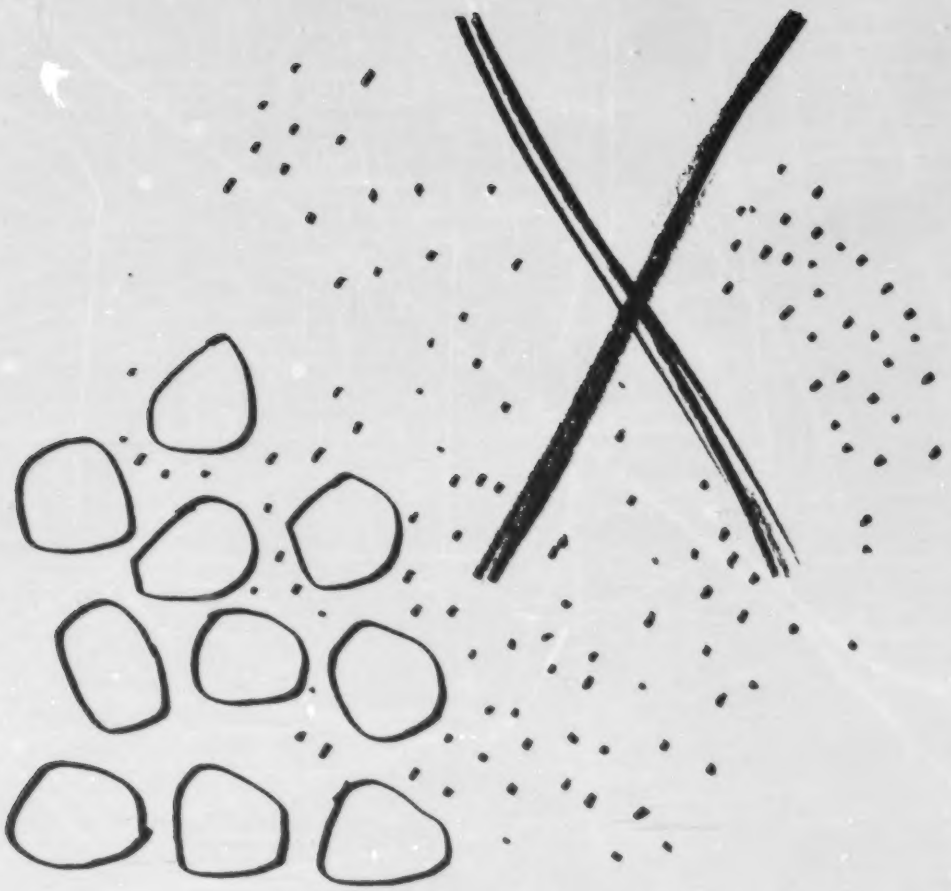
THE ROMAN BOY

Sometime later I traveled across an ocean on Earth and found a Roman boy counting colorful pebbles.

I asked him how many pebbles he had. He counted his pebbles for me as he scratched his numerals in the sand.

For every one pebble he scratched "1" in the sand

For every five pebbles he scratched "V" in the sand. And . . .



for every ten pebbles he scratched an "X" in the sand. They were his kind of numerals or symbols.

I saw this picture in the sand: X X X V. I read the numerals. This is what I said, '10, 20, 30, and one V means you have 35 pebbles.' The Roman boy said I was right.

After that I went flying away from the light of Earth, through the silver gray clouds into my space place up here. I have been here ever since.

Worksheet 11 Name:_____

ROMAN NUMERALS I

I. Can you read these?

Key: I = 1 V = 5 X = 10

1) XI = _____ 5) XVII = _____

2) XX = _____ 6) VI = _____

3) XV = _____ 7) VIII = _____

4) XXXII = _____ 8) III = _____

II. Find the answers. Write them in Roman numerals.

1) III - II = _____

2) XII - II = _____

3) VII - I = _____

4) VII - V = _____

5) XX + X = _____

6) V + I = _____

7) X + V + I =_____

8) X + X + X V + II =_____

Worksheet 12
ROMAN NUMERALS II

Name:_____

The Romans wrote I, II, III.

They wrote V.

When they wrote six they added I to the right of V.

It looked like this: VI.

Nine is one less than X.

How would you write nine? _____

Now try:

four	4	IV
nine	9	IX
twenty-nine		
	34	
		XXIV
nineteen		
	14	
thirty-nine		

Worksheet 13a
ROMAN NUMERALS III

Name:_____

Sometimes Roman numerals are used on clocks like this:

To talk about: What does VIII mean?

Why?

What does XII mean?

Why?

Worksheet 13 b
ROMAN NUMERALS III

Name:_____

1. Put the number symbol near a dot.

2. Show the time you come to school.

3. Make more clocks on another sheet of paper if you wish.

4. Tell what they show.

PART IV

THE AMERICAN BOY

The Space boys thought the old man had a wonderful adventure on Earth. They asked him question after question.

The kind old man said, "I think that you are ready to make your own visit to Earth. Go and see if you can meet an Earth child. Maybe you can learn something special about his kinds of treasures. Be sure that you tell me about your trip when you return."

The next day the boys decided to go to Earth to visit the land children whom the old man had told them about.

It was easy for them to make the trip because they were young and strong.

As they came toward Earth it looked big and bright.

After the boys flew through the clouds they landed on a place called "America." The Space boys were resting on a garden bush when they saw an American boy walking up and down rows of green plants looking for something.

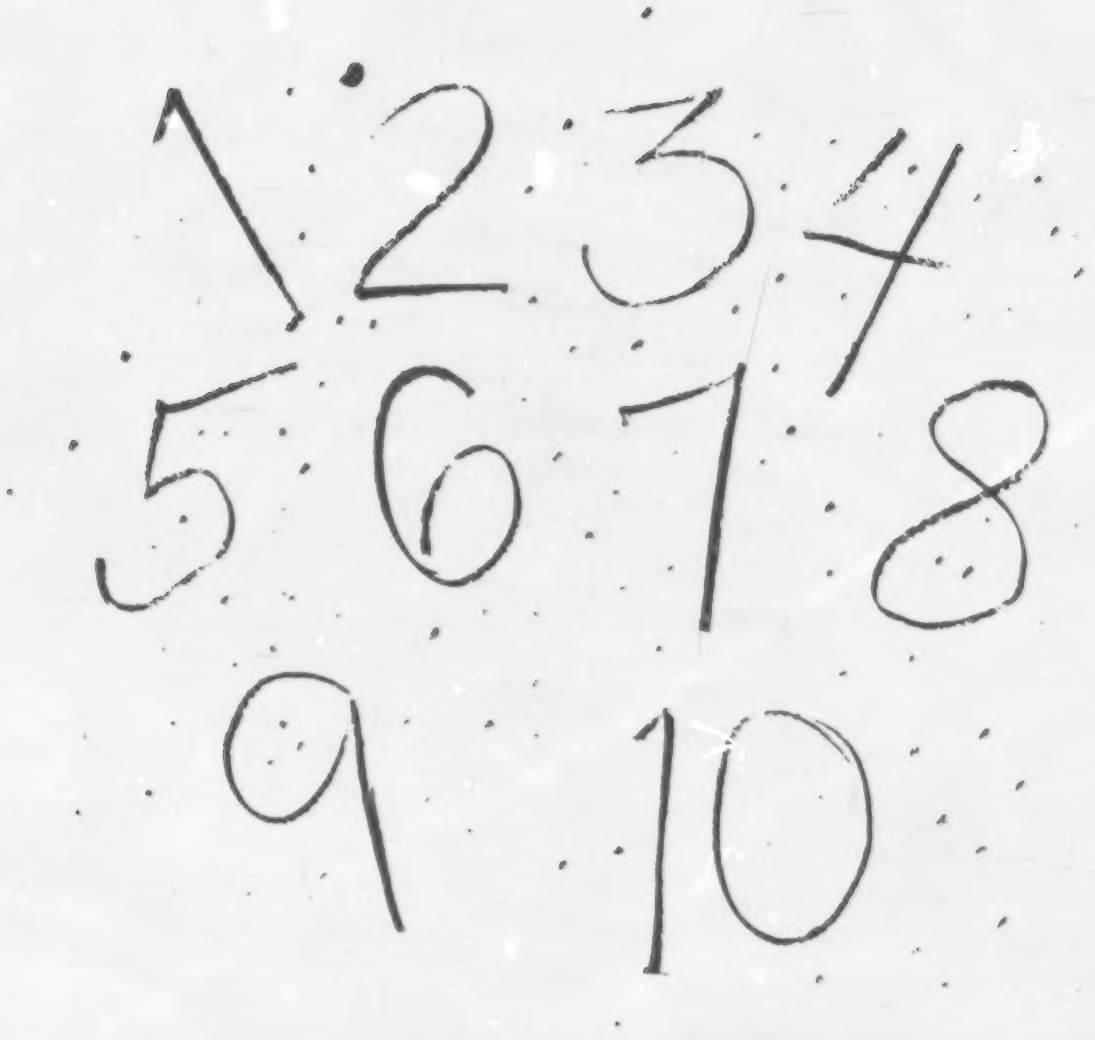
THE AMERICAN BOY

The Space boys asked the American boy if he had ever found a treasure on earth.

"Oh, yes," said the American boy. "I find my treasures here in this garden. My treasures are orange with black spots and they crawl. I keep them in this covered glass jar with holes in the top."

"How many lady bugs do you have?" asked the Taotians.

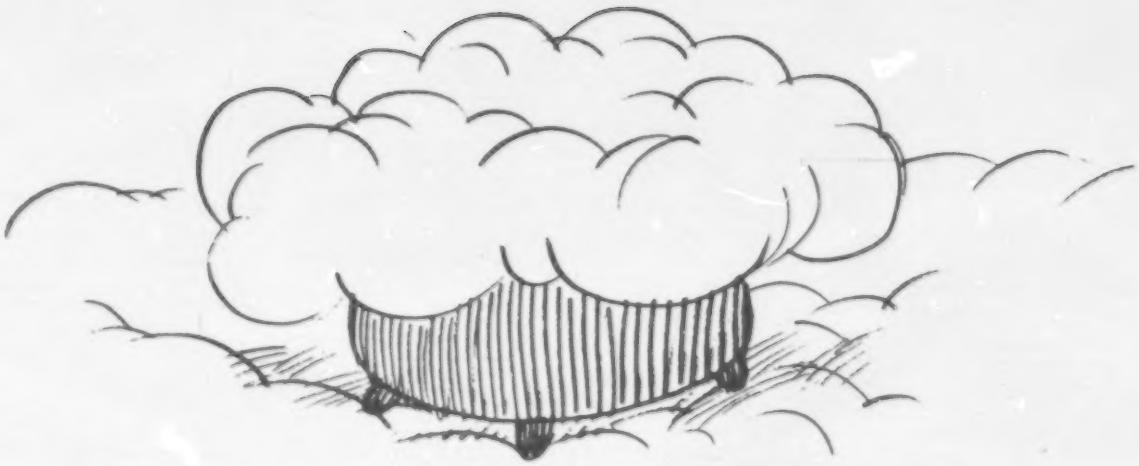
"I will count them for you," said the American boy. "As I count them I will scratch my kind of numerals in the black garden dirt with my finger."



He counted and scratched these numerals: "1, 2, 3, 4, 5, 6, 7, 8, 9, 10. I have ten ladybugs," said the American boy.

"Thank you for telling us about your ladybug treasure. Now we must be off to our planet Taos, again. Thank you for showing us your way of writing numerals."

The Space boys flew back and told their kind old man friend about the American boy and his ten ladybugs. They also told him about the way the American boy wrote his numerals. They told him that each numeral was written in a different way.



Then the Taotian boys went back to where they had left their treasure.

They took the cloud blanket off their golden coins and tucked it underneath the heavy treasure so it wouldn't float away.

They decided that they had the best treasure anywhere because it was full of coins which were so round and shiny that they looked like 75 little golden suns. And the treasure was theirs to keep forever.

There are many ways to express a number in written form. For example:

eight	VIII	8	10-2	6 + 2

In addition to these symbols, Minnemast introduces the "T" for ten. Numeral cards showing this notation are provided. The numbers are written in this manner:

ten	T
eleven	T + 1
twenty	2T
fifty-four	5T + 4

When this lesson is presented make the children understand that the "T" represents a set of ten and then there may be additional "ones" left over.

This work and understanding eases the way for the technique of grouping which follows in the next unit of work.

NOTE: For the worksheet "Writing Larger Numbers" dictate any numbers above ten that your class is able to handle.

Worksheet 14

IF "T" IS A SYMBOL FOR TEN Name:_____

"T" is a symbol for ten.
Draw a circle around a set of ten. How many do you have?

Write a symbol to show this.

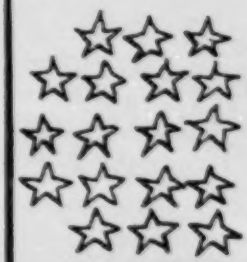
T+2

1.

2.

3.

4.



Worksheet 15

WRITING LARGER NUMBERS Name:_____

Use "T" for ten.

Then write the numbers your teacher says for you.

a.

b.

c.

d.

e.

f.

g.

h.

i.

j.

Worksheet 16

IN HOW MANY WAYS CAN YOU WRITE Name:_____

10	ten	6 + 4	11 - 1	X

1) four				
2) 6				
3) T + 2				
4) 3 + 2				
5) eleven				
6) 15				

USING THE NUMBER LINE

Materials:

Yardstick

Chalkboard

Chalk

Individual number line

Individual Follow-up Sheets

Vocabulary to be developed:

number line

trip

unit of measure

Conclusions to be drawn:

1. A number line may be used to illustrate a social need.
2. A number line has a point of origin usually labeled "0".
3. A number line consists of a series of uniform units of measure.
   These are numbered. The positive integers are associated with points to the right of "0". The negative integers are associated with points to the left of "0".
   Addition may be done by moving the required number of units along the number line.

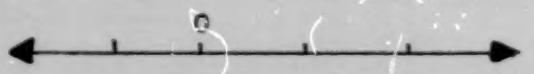
Background Information:

The number line is to be presented as one device which enables the child to understand counting and addition. The number line has other uses but these are not introduced at this time.

The number line is a geometric representation of the real number system, each point on the line is identified with one

and only one number.

The number line is straight and extends in either direction from a point of origin labeled "0". Arrows are used to show that a number line extends indefinitely in either direction.



Because a line is infinite, only a portion of it is reproduced and considered at a time:

A number line may be any line:

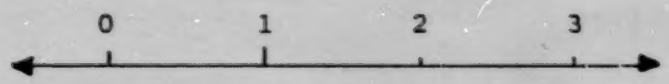
At this time, the horizontal position will be stressed. This direction is used in reading and is familiar to the child.

In the kindergarten and first grade the child has become acquainted with positive integers.

The integers are based on the need to count or measure whole units from a fixed point of origin in one of two directions:

The integers may be represented as geometric associations with a point on the line.

The point of origin is selected and labeled as "0". From this point, moving to the right, along the line, is the positive direction. These points are associated with the positive integers:

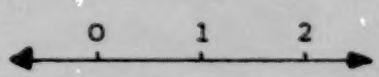


The negative integers are counted from the point of origin, "0", toward the left. The negative integers are not dealt with at this time, but may be recognized at the teacher's discretion and the pupils' discovery.

Procedure

Present the number line concepts in story form as trips or steps along a given line. Or you may wish to be more factual if your class would prefer this type of motivation.

Be sure that the child counts the segments or units of measure as distance from the "0" point in this manner:

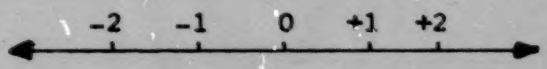


In class demonstration, acquaint the children with these symbols 9 ⟶ 15 as meaning, begin at 9 and move to the right to 15. Likewise, 5 ⟶ 11, indicates beginning at 5 and moving right to 11. 5 ⟵ 11 indicates beginning at 11 and moving left to 5.

A NUMBER LINE

1. A number line is a straight line.
2. A number line is marked off in units of measure. (Introduce this phrase, units of measure, as the occasion permits.)

3. The unit of measure is marked off by a point.
4. A point is chosen as a starting point and it is identified with zero.
5. The points other than zero are numbered in order from zero extending to the right.
6. The numbers to the right of zero on the number line are called positive numbers.
7. A number line has points to the left of zero. These numbers to the left of zero are called negative numbers.
8. The number line extends indefinitely to the left and right. This is shown by the use of arrows.

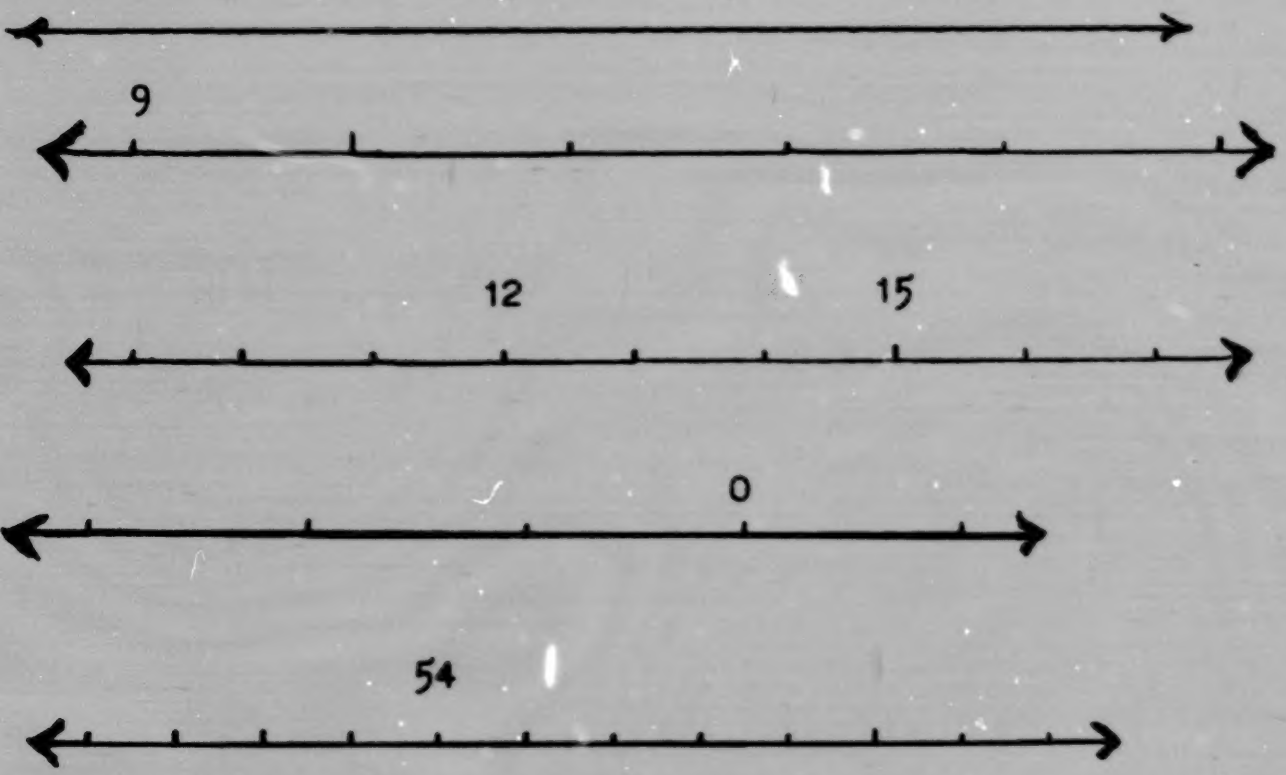


Worksheet 17

LABEL A NUMBER LINE Name: _____

These are parts of a number line.

Can you write the numeral above the indicated points?



Worksheet 16

MOVES ON THE NUMBER LINE Name:_____

Example:

a. The frog jumped 4 steps to the right.
Color them green.

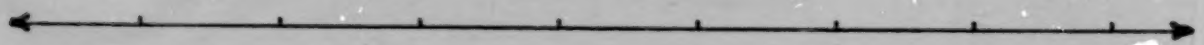
Now underline these:

1. The kangaroo jumped 3 big hops to the right.
Color them orange.

2. The horse took 6 big gallops to the left.
Color them brown.

3. The rabbit made 9 hops to the right.
Color them red.

4. The ball bounced 7 times to the right.
Color them blue.



5. Jane skipped 5 skips to the left.
Color them yellow.

Worksheet 19

HOW MANY UNITS DO YOU MOVE?

Name: _____

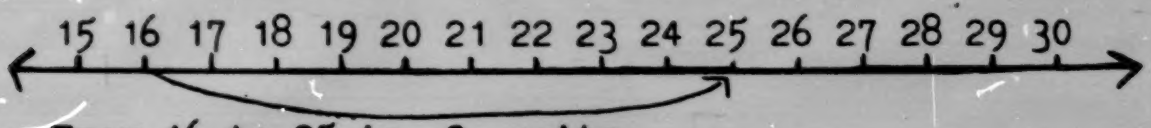
These are number lines.

Two points on the number line are given.

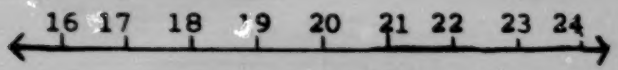
How many units apart are they?

You may move in either direction on the number line.

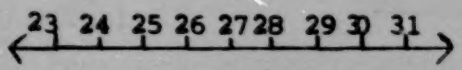
Do it like this:



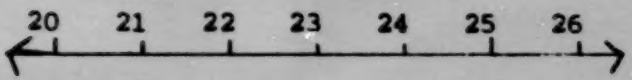
From 16 to 25 is 9 units.



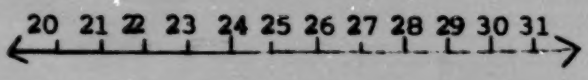
From 16 to 22 is ___units.



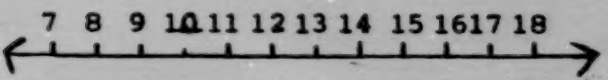
From 24 to 30 is ___ units.



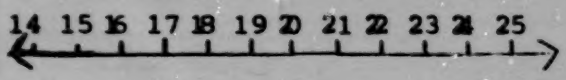
From 26 to 20 is ___ units.



From 30 to 20 is ___ units.



From 16 to 9 is ___ units.



From 15 to 23 is ___ units.

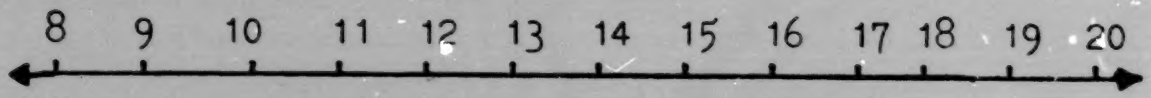
Worksheet 20

WORKING WITH A NUMBER LINE

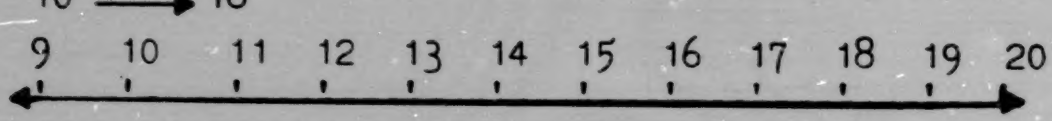
Name: _____

Show your moves.

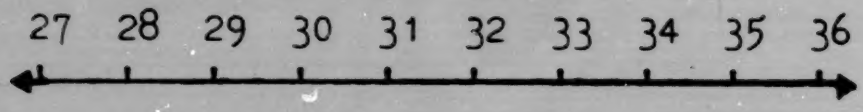
9 → 15



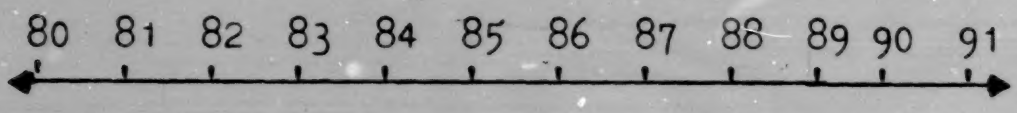
10 → 18



29 → 35



83 → 90



Worksheet 21

WORKING WITH A NUMBER LINE

Name:_____

Use your number line to find how many units you must move between these numbers. Move in the correct direction.

8 → 17	73 → 60
19 → 23	54 → 62
45 → 54	91 → 99
7 → 13	33 → 40
6 → 15	18 → 26
62 → 75	29 → 37
28 → 34	89 → 93

Worksheet 22

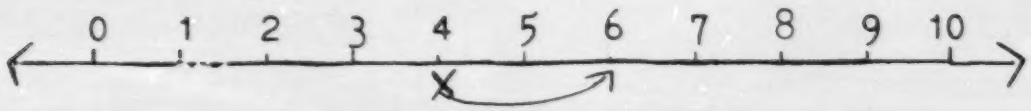
ALIBI

Name: _____

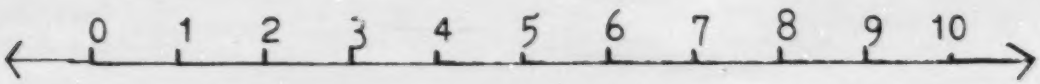
These are parts of num er lines.

Show how <u>the place</u> on a number line has changed.

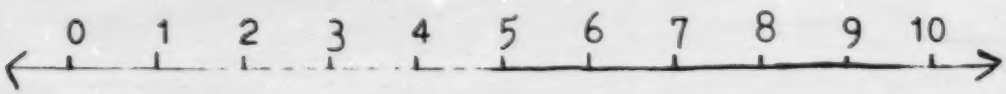
Do it like this:



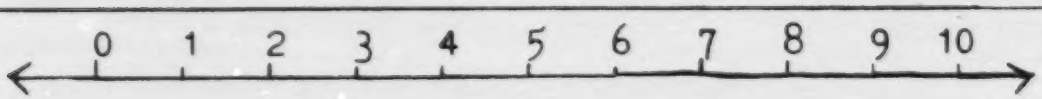
4 + 2 = 6



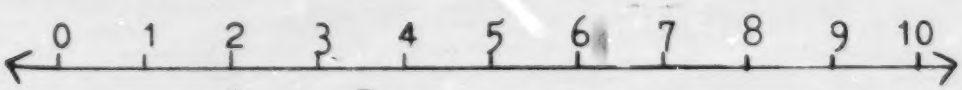
5 + 3 = ____



7 + 3 = ____



4 + 5 = ____



2 + 7 = ____

Worksheet 23

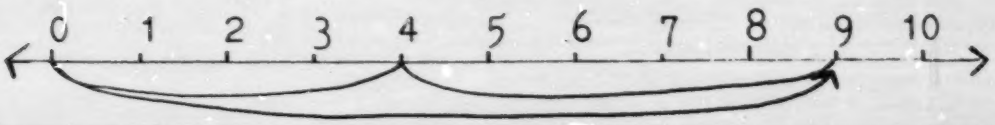
A SINGLE MOTION

Name: _____

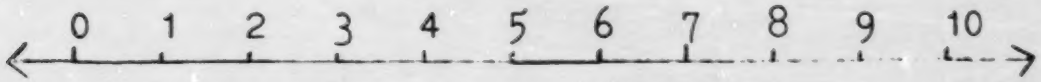
Can you do this?

Show what single motion is the same as

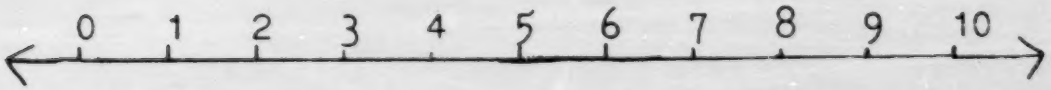
4 to the right followed by 5 to the right



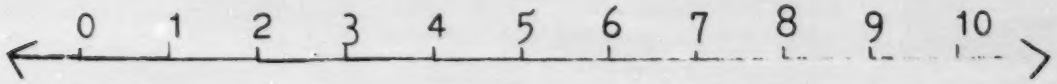
5 to the right followed by 4 to the right



4 to the right followed by 2 to the right



6 to the right followed by 4 to the right

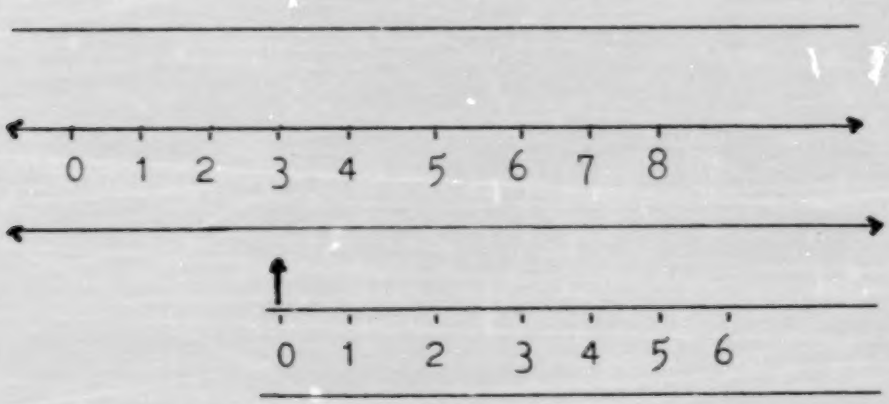


Worksheet 24
ALIAS I

Name: _____

Alias addition is finding another name.
Another name for 3 + 5 is 8.

Use two scales. To find another name for 3 + 5 place them like this:



Use the scales to find another name for:

5 + 2 is ____ 3 + 3 is ____

2 + 4 is ____ 3 + 2 is ____

4 + 3 is ____ 4 + 4 is ____

5 + 4 is ____ 6 + 3 is ____

Worksheet 25

ALIAS II

Name: _____

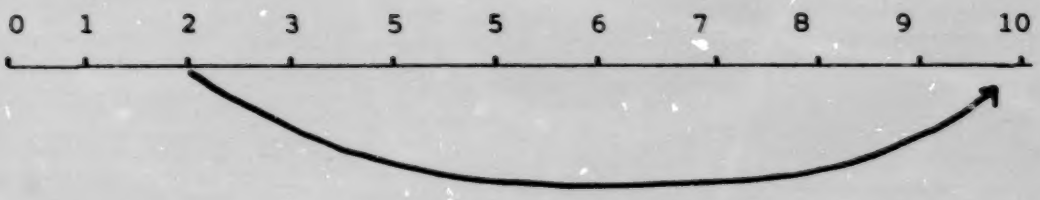
Use the scales to find another name for the number given below:

9 + 3 = ____	4 + 8 = ____
6 + 4 = ____	5 + 6 = ____
8 + 3 = ____	3 + 7 = ____
7 + 5 = ____	5 + 4 = ____
6 + 6 = ____	2 + 6 = ____
3 + 7 = ____	2 + 9 = ____

FINDING THE MISSING ADDEND*

Examples such as 10 - 2 = ____ are equivalent to 2 + ___ = 10. If the children are led to see this relationship the number line can be used in the same manner as it was in addition.

10 - 2 = ___
2 + __ = 10



a move of 8

Eight is the missing addend.

This is a very simple procedure for the child. Examples should be written as

10 - 2 = ___ , 2 + ___ = 10, ___ + 2 = 10

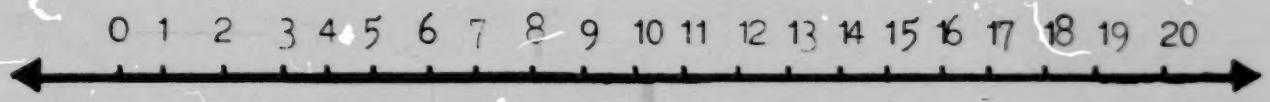
so that the child recognizes that a missing addend is needed.

*Revision of subtraction should be carried out from Unit VIII, page 42.

Worksheet 26
FIND THE MISSING ADDEND

Name:_____

Use the number line to find the missing addend.



1.) 15 - 4 = ____

2.) 5 + __ = 20

3.) __ + 3 = 12

4.) 10 - 8 = ___

5.) ___ + 7 = 14

6.) 18 - 7 = ___

7.) 12 + __ = 19

8.) 16 - 7 = __

9.) 9 + __ = 15

10.) 20 - 3 = __

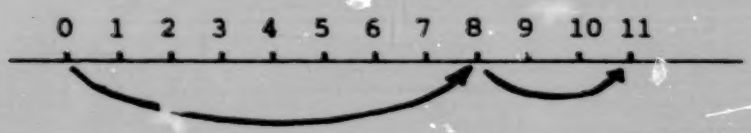
11.) 4 + __ = 18

12.) __ + 6 = 13

WORKING WITH THREE ADDENDS

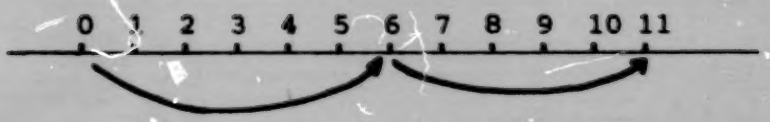
Interest in the number line may be maintained by introducing examples having three addends such as 6 + 2 + 3. Lead the children to see that they must determine the result of two addends first and then add the third. This is shown in this manner.

(6 + 2) + 3



or

6 + (2 + 3)



This is known as the associative principle in mathematics or the law of grouping. The addition in the parentheses must be done first. Have the class draw their own conclusion as to what happens if they group in one or the other way.

Worksheet 27

WORKING WITH THREE ADDENDS Name:_____

Group the addends.

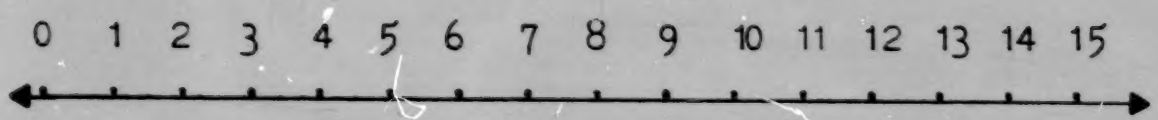
Then find the sum.

Use a number line to help you.

Example: $3 + 2 + 4 =$

$(3 + 2) + 4$ or

$3 + (2 + 4) = 9$



1.) $6 + 3 + 4 =$ ____

2.) $5 + 5 + 3 =$ ____

3.) $2 + 7 + 3 =$ ____

4.) $8 + 1 + 3 =$ ____

5.) $4 + 4 + 3 =$ ____

6.) $7 + 2 + 1 =$ ____

7.) $7 + 7 + 1 =$ ____

8.) $4 + 5 + 3 =$ ____

9.) $6 + 7 + 2 =$ ____

10.) $8 + 4 + 3 =$ ____

11.) $2 + 3 + 5 =$ ____

12.) $3 + 7 + 4 =$ ____

13.) $4 + 8 + 3 =$ ____

14.) $6 + 6 + 3 =$ ____

RELATIONSHIP OF NUMBERS AS EXPRESSED BY GREATER THAN, LESS THAN, AND EQUAL TO

Materials:

number line

Background:

The child should have numerous experiences in noting relationships which exist among numbers. He should understand that when two numbers are compared there are three possibilities: x equals y ($x = y$), x is greater than y ($x > y$), or x is less than y ($x < y$).

The first exercises which follow are restricted to the use of the symbol for "greater than," ($>$). Exercises using the symbol for "less than" ($<$) are introduced separately in the next set of exercises. The last set includes all three symbols: equal ($=$), greater than ($>$), and less than ($<$).

Examples:

$3 > 2$ Three is greater than two.

$2 < 3$ Two is less than three.

$2 + 3 = 5$ Two plus three equals five.

Procedure:

Today we are going to compare numbers. Let's consider sixteen and ten. Which is greater? Here is a simple way to write that sixteen is greater than ten:

$16 > 10$

We read it: sixteen is greater than ten.

Which is greater, fifteen or twenty? Can you write this in a simple way not using written words now? (Call on a class member to write the statement on the board using the symbol for greater than as $20 > 15$.)

I'll write a few more pairs of numbers on the board for you to compare. Decide which of the two in each pair is the greater, then be prepared to come to the board and write the statement telling which is greater using the new symbol.

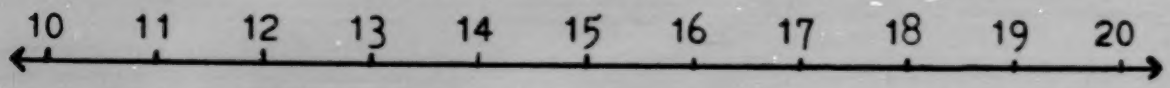
15, 13	46, 59	92, 98
23, 30	80, 40	33, 65

Worksheet 28

AN EASY WAY TO WRITE "GREATER THAN"

Name:_____

Here is a part of the number line:



Sometimes symbols help us to write something in an easy and quick way.

This symbol means "greater than" >.

Write the following pairs of numbers and a symbol so that one is expressed as being greater than the other.

16, 14 16 > 14	17, 12 _____ > _____	18, 19 _____ > _____
16, 11 _____ > _____	11, 12 _____ > _____	13, 10 _____ > _____
10, 15 _____ > _____	14, 18 _____ > _____	16, 19 _____ > _____

Unit XI

"LESS THAN"

Procedure:

Let's compare numbers in a way other than which is the greater. Which is less, five or nine? Here is a simple way to write that 5 is less than 9.

$5 < 9$

We read it: five is less than nine.

Which is less, 10 or 15? Can you write this in a simple way using the symbol for less than?

I'll write a few more pairs of numbers on the board for you to compare. Decide which of the two in each pair is less than the other, then be prepared to come to the board and write the statement telling which is less using the symbol.

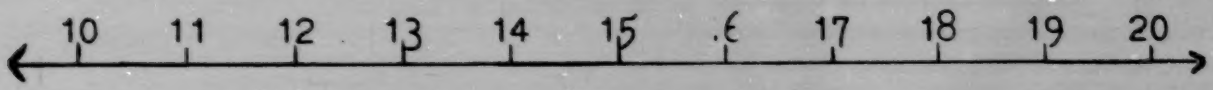
18, 24	9, 8	99, 1
21, 12	31, 43	7, 22

Worksheet 29

AN EASY WAY TO WRITE "LESS THAN"

Name:_____

Here is a part of the number line:



Sometimes symbols help us to write something in an easy and quick way.

This symbol means "less than" $<$.

Write the following pairs of numbers so that one is expressed as being less than the other.

17, 12	14, 17	12, 18
_____ < _____	_____ < _____	_____ < _____
16, 14	18, 19	16, 20
_____ < _____	_____ < _____	_____ < _____
11, 12	15, 13	19, 12
_____ < _____	_____ < _____	_____ < _____

Worksheet 30
GREATER THAN OR LESS THAN

Name:_____

GREATER THAN OR LESS THAN

Put in the sign for "greater than" or "less than"

16 ___ 14	17 ___ 12	18 ___ 19
16 ___ 11	11 ___ 12	13 ___ 10
10 ___ 15	14 ___ 18	16 ___ 19

Worksheet 31

GREATER THAN, LESS THAN, EQUAL TO

Name: _____

Greater than	$3 + 2 > 4$
Less than	$9 - 2 < 8$
Equal to	$4 + 3 = 7$

Put in the symbol to make a true statement:

1) $9 + 3 ___ 10$

2) $6 + 2 ___ 7$

3) $8 - 5 ___ 4$

4) $7 - 3 ___ 4$

5) $10 - 2 ___ 7$

6) $8 + 3 ___ 11$

7) $6 - 3 ___ 4$

8) $5 - 2 ___ 3$

9) $10 + 5 ___ 16$

10) $5 + 4 ___ 8$

Worksheet 32

GREATER THAN, LESS THAN

Name:_____

Greater than $9 - 2 > 8 - 3$

Less than $(2 + 3) + 1 < (2 + 2) + 3$

Put in the symbol to make a true statement:

1. $3 + 6$ _____ $4 + 2$
2. $4 + 3 + 1$_____ 9
3. $5 + 2$ _____ $9 - 4$
4. $7 + 3$ _____ $2 + 9$
5. $3 + 9$ _____ $(4 + 4) + 3$
6. $7 + 5$_____ $(6 + 6) + 1$
7. $8 - 6$_____ $3 + 1$
8. $(5 + 5) + 2$_____ 13
9. $8 + 4$ _____ 16
10. $(9 + 2) + 1$_____ 11

DETERMINING SUMS BY MEANS OF RELATIONSHIPS OF NUMBERS

Children can see relationships between larger numbers. They can determine the sums of two addends by recognizing relationships which exist between two given numbers.

For example have them complete this row of sums:

	10	11	12	13	14	15	16	17	18	19
13					27					

Procedure:

I have added the 13 and 14 and have written the sum 27 on the chart.

Who can add 13 and 13 and write the sum in the proper square?

(Fill in the chart.)

Let us try another.

(Be sure to do enough so that even the slowest child grasps the relationship without having to be told.)

	17	18	19	20	21	22	23	24
23			42					

(Keep the numbers large so that the child is forced to recognize relationships and does not know the fact.)

Worksheet 33

SEEING RELATIONSHIPS I

Name: _____

Find the sum of a pair of numbers. One number is in the top row. The other number is in the first column.

	30	31	32	33	34
24			56		
25				58	
26		57			
27				60	
28		59			

The pair of addends are 24, 32 and the sum is 56.

The pair of addends are 24, 33, and the sum is _____.

Find the sums for that row.

Now finish the chart.

Do you notice anything about it?

Worksheet 34

SEEING RELATIONSHIPS II

Name: _____

Now find these sums:

1.

	58	59	60	61
39			99	
40				101
41	99			
42			102	

2.

	76	77	78	79
57				136
58	134			
59		136		
60			138	

"BETWEEN" - A WAY TO ESTIMATE

Materials:

Rulers available at Minnemast

Purpose:

It is valuable for the child to be able to estimate sums. He may desire to predict an approximate sum for utilitarian purposes. He may want to make a quick check to determine if an answer is reasonably correct.

Procedure:

Estimating is presented by manipulating two number lines. In the second grade, the rulers available at Minnemast are used because they have a minimum number of divisions. Use the "inch" side. Give each child two rulers or group children so that each group has 2 rulers with which to work.

Introduction:

Mother was going shopping. She planned to buy a dress for Judy which would cost between three and four dollars, and a pair of shoes for Bill which would cost between five and six dollars.

Some questions about the number of dollars she needed raced through Mother's mind. I shall write these on the board for you. On the board write:

Between 3 and 4 dollars for a dress
Between 5 and 6 dollars for shoes

1. What is the least amount of money I could spend?
2. What would be the most?

Can anyone think of the answer to the questions?
Here is a way to find the answers.

Some approximations for the children to make are:

If you add a number between _____and_____to a number between ____and_____ tne sum will be between _____and_____.

---

2 and 3, 4 and 5

5 and 6, 2 and 3

6 and 7, 3 and 4

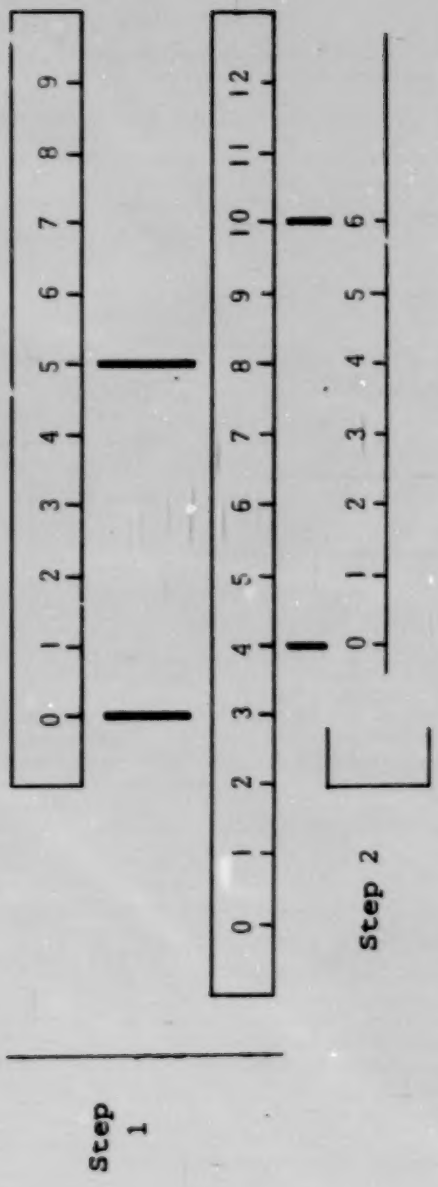
4 and 5, 4 and 5

8 and 9, 3 and 4

6 and 7, 5 and 6

Example:

If we add a number between 3 and 4 to a number between 5 and 6 the sum will be between what two numbers ?



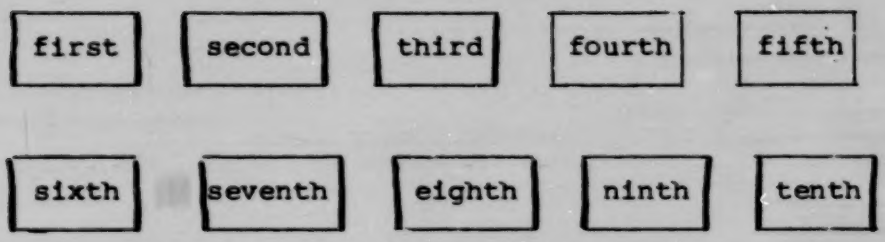
To find a sum "between" two numbers:

1. Take 2 rulers.
2. Place one above the other with the inch side facing you.
3. Align the top ruler with the lowest number, 3.
4. On the top ruler locate the corresponding greater number, 5.
5. Read the number on the lower ruler which is aligned with the 5. Follow the arrow to 8.
6. Repeat this process of alignment as shown in the lower scale to locate the ten.

ORDINAL NUMBERS

Materials:

Word cards labeled:



Purpose:

To learn the ordinal numbers for the first ten cardinal numbers.

Procedure:

1. Place a few objects before the class. Ask someone to bring you the third, fifth, etc.
2. Have someone tag the child in a row. (seventh, ninth, etc.)
3. Have a race. As you call the number the child in that row stands. (second, fourth, etc.)
4. Present the words as for reading. You may wish to do a few a day.

Continue with the worksheets.

Worksheet 35

ORDINAL NUMBERS I

Name: _____

Directions: Start from the left and go to the right.
Put a circle around the correct object.

second

fourth

third

first

fifth

Worksheet 36

ORDINAL NUMBERS II

Name:_____

Directions: Start at the right and go to the left.
Put a circle around the correct object.

fifth

tenth

eighth

seventh

ninth

Procedure:

Explain to the children that the five drawings on their worksheet show pictures of five skyscrapers and that each box represents another floor of the skyscraper. Have them count the number of floors in the tallest and shortest building and compare the others.

Direct the class to put certain names on certain floors as you read to them the following directions.

1. Jane lives on the tenth floor of a skyscraper. Put Jane's name where she lives.
2. Mary lives on the third floor of a skyscraper that is seven floors high. Put Mary's name where she lives.
3. Tom lives on the top floor of a skyscraper. He lives on the eighth floor. Put Tom's name where he lives.
4. John lives on the first floor of a skyscraper with eight floors. Put John's name where he lives.
5. Jim lives two floors above John. Put Jim's name where he lives.
6. Patty lives on the middle floor of a skyscraper with five floors. Put Patty's name where she lives.
7. Billy lives in a skyscraper with six floors. He lives on the top floor in that skyscraper. Put Billy's name where he lives.

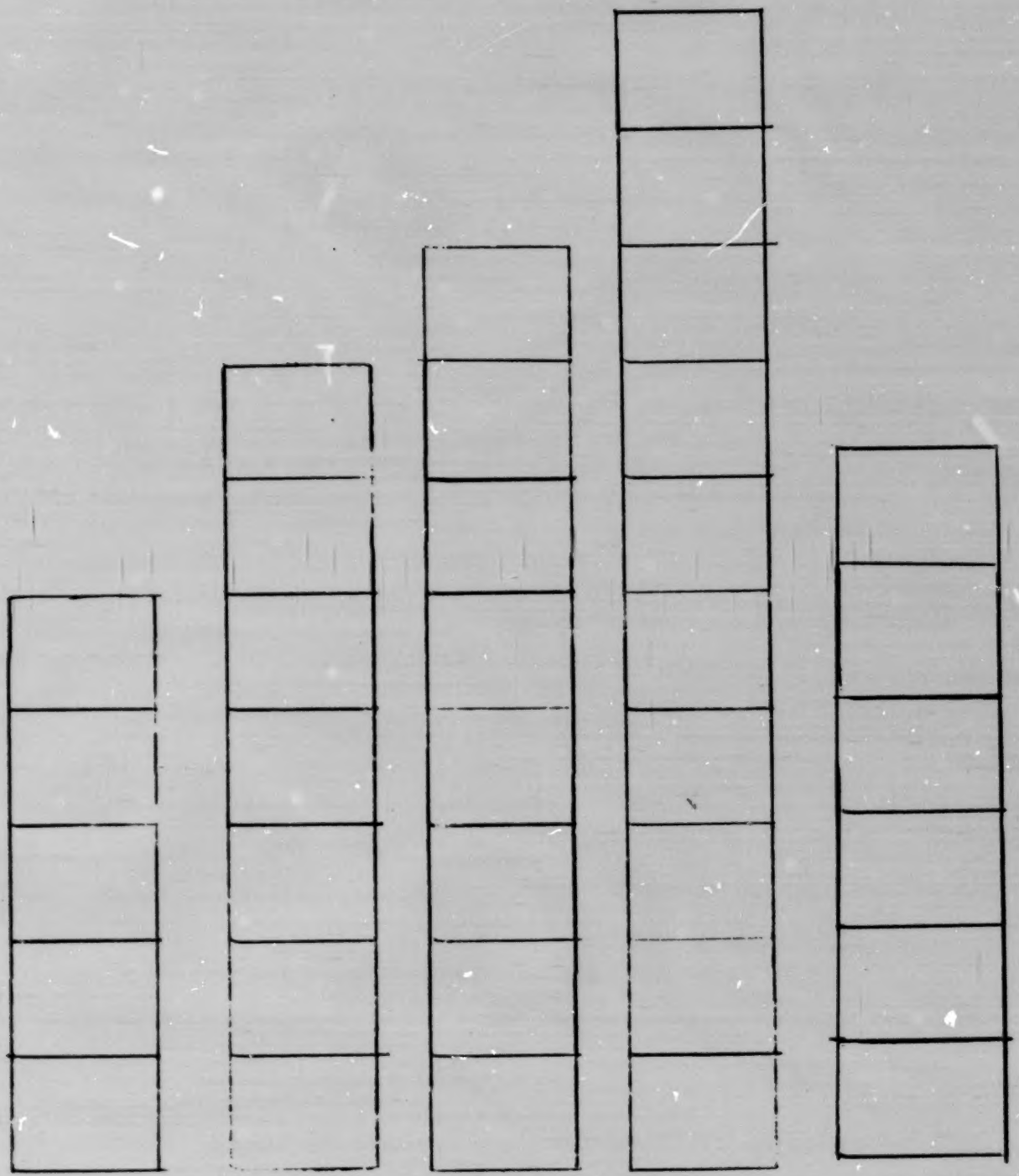
As a culmination activity, have the word cards, (first, second, third, fourth, fifth, sixth, seventh, eighth, ninth, and tenth) available for the children to hold up as you name various boys and girls who live on the floors of the skyscrapers.

Example: "Which floor does Jane live on?" (tenth)

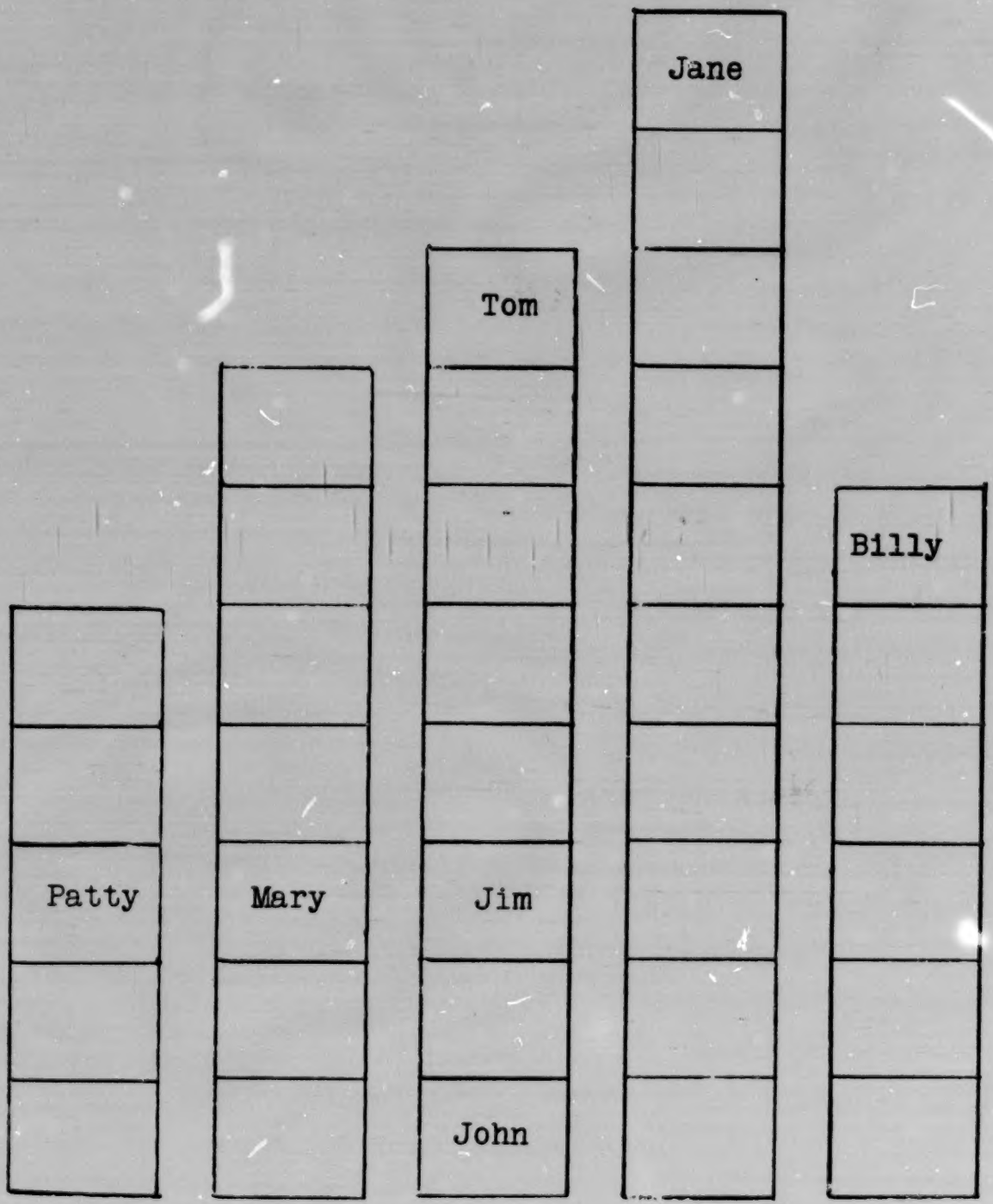
Worksheet 37

A SKYSCRAPER Name:_____

Jane - John - Mary - Jim - Patty - Billy - Tom



Answer Sheet



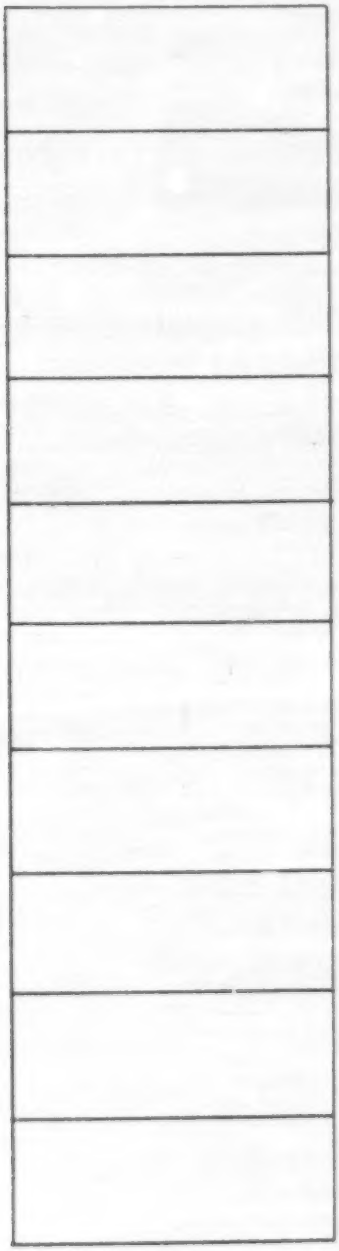
Worksheet 38

ANOTHER SKYSCRAPER

Name: _____

This is a Skyscraper:

Put the right name on each floor.



1. Jane lives on the fourth floor.
2. John lives on sixth floor.
3. Joe lives on tenth floor.
4. Mary lives on first floor.
5. Bob lives on eighth floor.
6. Patty lives on third floor.
7. Jim lives on ninth floor.
8. Billy lives on seventh floor.
9. Timmy lives on fifth floor.
10. David lives on second floor.